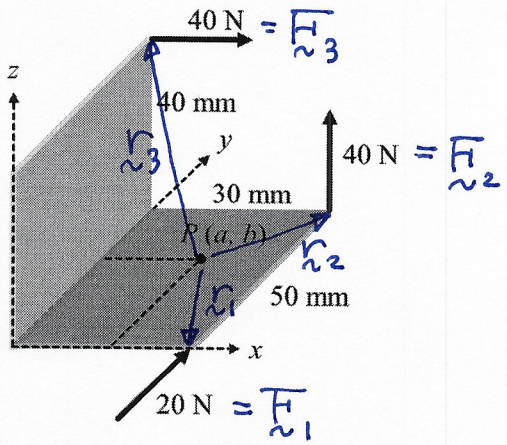
정역학(Statics): 중간고사(Mid-Term Exam)

2020. 10. 26. 11:00~13:00, Closed Book
Instructor: 최승태 교수

학번(Student ID):_____ 이름(Name): 최승태 좌석번호(Seat No):____

1. (20점) 그림에 주어진 L자형 구조물에 작용하는 힘에 대한 Wrench 등가힘계(equivalent force system)를 구하시오. 또한 Wrench 등가힘계의 작용선(line of action)이 xy 평면과 교차하는 점 P의 xy 평면 상의 좌표 (a, b)를 구하시오.



④ $\underset{\sim}{R} = 40\underset{\sim}{i} + 20\underset{\sim}{j} + 40\underset{\sim}{k}$ (N)

$\underset{\sim}{r}_1 = (30-a)\underset{\sim}{i} - b\underset{\sim}{j}$, $\underset{\sim}{F}_1 = 20\underset{\sim}{j}$

$\underset{\sim}{r}_2 = (30-a)\underset{\sim}{i} + (50-b)\underset{\sim}{j}$, $\underset{\sim}{F}_2 = 40\underset{\sim}{k}$

$\underset{\sim}{r}_3 = -a\underset{\sim}{i} + (50-b)\underset{\sim}{j} + 40\underset{\sim}{k}$, $\underset{\sim}{F}_3 = 40\underset{\sim}{i}$

⑧ $\underset{\sim}{M}_P = \underset{\sim}{r}_1 \times \underset{\sim}{F}_1 + \underset{\sim}{r}_2 \times \underset{\sim}{F}_2 + \underset{\sim}{r}_3 \times \underset{\sim}{F}_3$

$= 20(30-a)\underset{\sim}{k} - 40(30-a)\underset{\sim}{j} + 40(50-b)\underset{\sim}{i}$

$- 40(50-b)\underset{\sim}{k} + 1600\underset{\sim}{j}$

$= 40(50-b)\underset{\sim}{i} + 40(10+a)\underset{\sim}{j} + 20(2b-a-70)\underset{\sim}{k}$

④ $\underset{\sim}{R} // \underset{\sim}{M}_P$ (Wrench) 이므로,

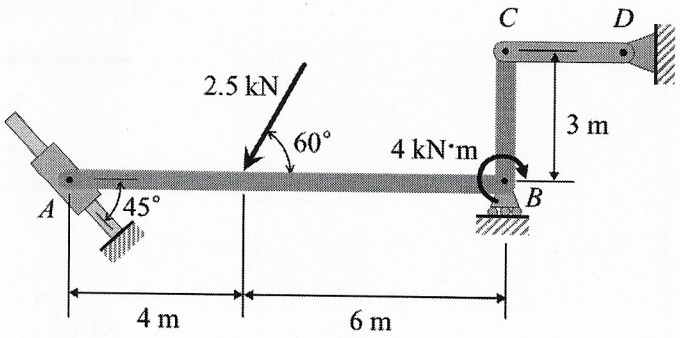
$(100-2b) : (20+2a) : (2b-a-70) = 2 : 1 : 2$

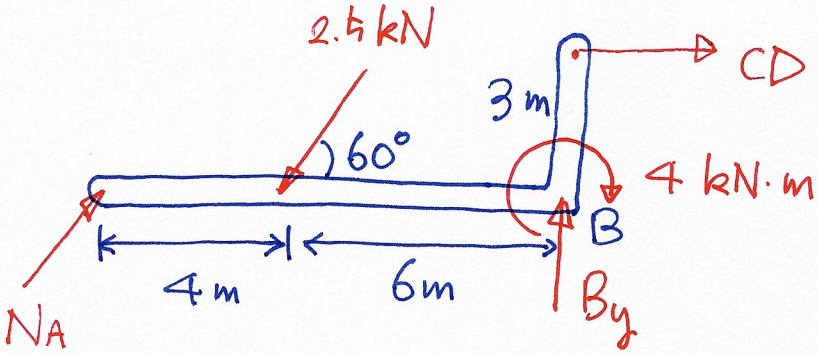
④ $\therefore (a, b) = \left(-\frac{50}{9}, \frac{370}{9}\right)$ (mm)

$\therefore \underset{\sim}{R} = 40\underset{\sim}{i} + 20\underset{\sim}{j} + 40\underset{\sim}{k}$ (N)

② $\therefore \underset{\sim}{M}_P = \frac{1600}{9}(2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k})$ (N·mm)

2. (20점) 그림에 주어진 구조물에 대하여 점 A, B, 및 C에 걸리는 반력을 구하시오. 점 A의 slide는 마찰이 없고, 모든 물체의 자중은 무시한다.)





(8)

$$\Sigma F_x = CD - 2.5\cos 60^\circ + N_A \cos 45^\circ = 0 \quad (2)$$

$$\Sigma F_y = B_y - 2.5\sin 60^\circ + N_A \sin 45^\circ = 0 \quad (2)$$

$$\Sigma M_B = 2.5\sin 60^\circ \times 6 - 4 - CD \times 3 - N_A \sin 45^\circ \times 10 = 0 \quad (2)$$

$$CD + \frac{1}{\sqrt{2}} N_A = 1.25 \quad \text{— ①}$$

$$3CD + 5\sqrt{2} N_A = 7.5\sqrt{3} - 4 \quad \text{— ②}$$

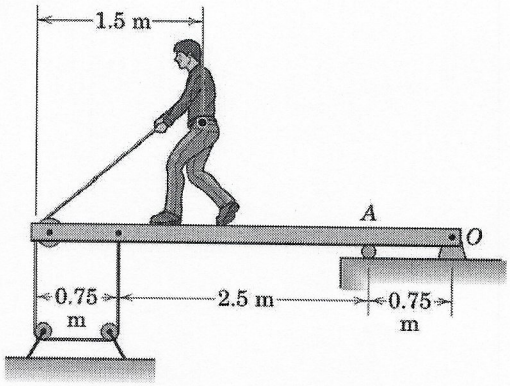
$$② - 3① \rightarrow \frac{7}{2}\sqrt{2} N_A = 7.5\sqrt{3} - 7.75$$

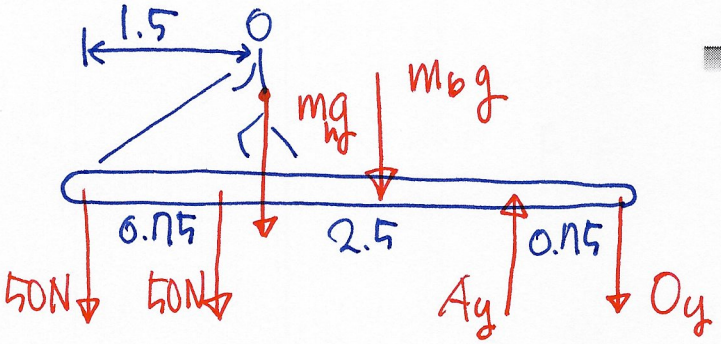
$$\therefore N_A = \frac{1}{7\sqrt{2}}(15\sqrt{3} - 15.5) \ (kN) \quad (2)$$

$$\therefore CD = \frac{1}{14}(33 - 15\sqrt{3}) \ (kN) \quad (2)$$

$$\therefore B_y = \frac{1}{28}(31 + 5\sqrt{3}) \ (kN) \quad (2)$$

3. (20점) 그림과 같이 균일한 100 kg의 보의 강성을 시험하기 위해 60 kg의 소년이 줄을 50 N으로 잡아 당기고 있다. 점 O에 걸리는 반력을 구하시오. 단, 중력가속도는 g이다.





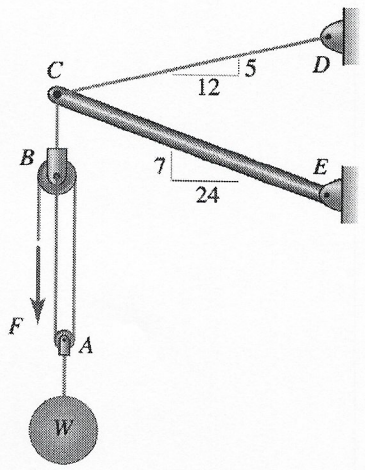
(7)

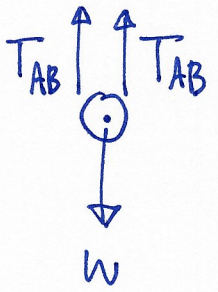
$$\Sigma M_A = -0.75 \times O_y + 2.5 \times 50 + 3.25 \times 50 + 1.75 \times m_g + 1.25 \times m_b g = 0 \quad (7)$$

$$3O_y = 1150 + 420g + 500g$$

$$\therefore O_y = \frac{1}{3}(1150 + 920g)\ \text{N} \quad (6)$$

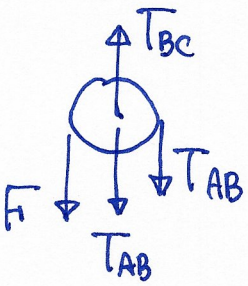
4. (20점) 그림과 같은 승강장치(Hoist)가 무거운 물체를 들어 올리는 데에 사용된다. 도르래 A 및 B 사이의 Cable은 1.2 kN을 지지할 수 있고, 다른 모든 cable은 2.2 kN을 지지할 수 있다. 봉 CE는 2.6 kN의 압축력을 지지할 수 있다. 이 승강장치로 들어올리 수 있는 최대 무게를 구하시오.





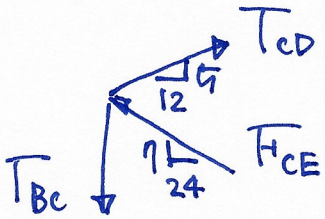
$\Sigma F_y = 2T_{AB} - W = 0$

$\therefore T_{AB} = \frac{1}{2}W$ ②



$F = T_{AB}$

$\Sigma F_y = T_{BC} - 3T_{AB} = 0 \quad \therefore T_{BC} = 3T_{AB} = \frac{3}{2}W$ ②



$\Sigma F_x = \frac{12}{13}T_{CD} - \frac{24}{25}F_{CE} = 0 \quad \therefore F_{CE} = \frac{25}{26}T_{CD}$

$\Sigma F_y = \frac{5}{13}T_{CD} + \frac{7}{25}F_{CE} - T_{BC} = 0$

$\frac{5}{13}T_{CD} + \frac{7}{26}T_{CD} = T_{BC}$

$\therefore T_{CD} = \frac{26}{17}T_{BC} = \frac{39}{17}W$ ②

$\therefore F_{CE} = \frac{25}{26}\frac{39}{17}W = \frac{75}{34}W$ ②

② In the cable AB, $T_{AB} = \frac{W}{2} \leqq 1.2\,kN \quad \therefore W \leqq 2.4\,kN$

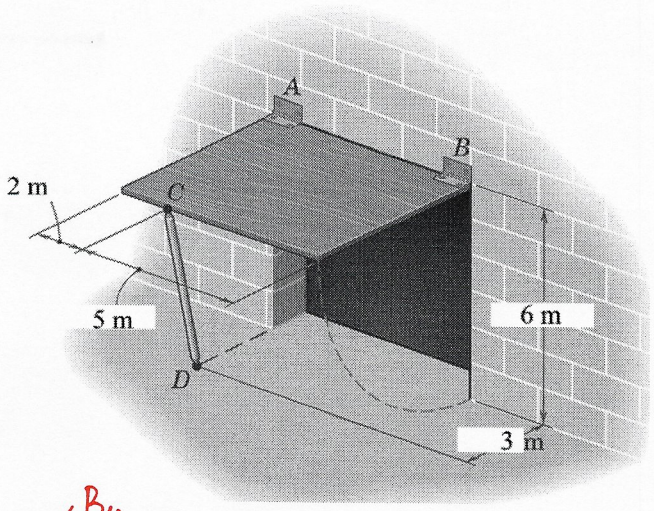
② In the cable BC, $T_{BC} = \frac{3}{2}W \leqq 2.2\,kN \quad \therefore W \leqq \frac{4.4}{3}\,kN$

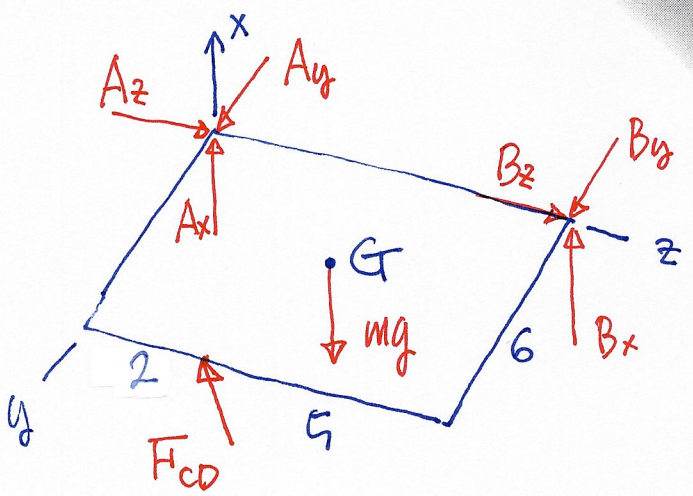
② In the cable CD, $T_{CD} = \frac{39}{17}W \leqq 2.2\,kN \quad \therefore W \leqq \frac{17 \times 2.2}{39}\,kN$

② In the bar CE, $F_{CE} = \frac{75}{34}W \leqq 2.6\,kN \quad \therefore W \leqq \frac{34 \times 2.6}{75}\,kN$

④ $\therefore W_{max} = \frac{17 \times 2.2}{39}\,kN$

5. (20점) 질량 100 kg의 직사각형 문이 단일 지지대 CD에 의해서 90° 열린 채로 고정되어 있다.
(1) 단일 지지대 CD에 걸리는 힘 F를 구하시오.
(2) 작은 hinge A와 B 각각에서 hinge axis AB에 수직한 방향으로 작용하는 힘의 크기를 구하시오.
(중력가속도는 g이다.)





④

$$\underset{\sim}{F}_{CD} = F_{CD}\,\underset{\sim}{n}_{CD} \quad \text{where} \quad \underset{\sim}{n}_{CD} = \frac{6\underset{\sim}{i}+3\underset{\sim}{j}+2\underset{\sim}{k}}{\sqrt{6^2+3^2+2^2}} = \frac{1}{7}\left(6\underset{\sim}{i}+3\underset{\sim}{j}+2\underset{\sim}{k}\right)$$

④

$$\Sigma \underset{\sim}{F} = A_x\underset{\sim}{i} + A_y\underset{\sim}{j} + A_z\underset{\sim}{k} + B_x\underset{\sim}{i} + B_y\underset{\sim}{j} + B_z\underset{\sim}{k} - mg\underset{\sim}{i} + F_{CD}\,\underset{\sim}{n}_{CD} = \underset{\sim}{0}$$

$$\therefore A_x + B_x - mg + \frac{6}{7}F_{CD} = 0$$

$$\therefore A_y + B_y + \frac{3}{7}F_{CD} = 0$$

$$\therefore A_z + B_z + \frac{2}{7}F_{CD} = 0$$

③

$$\Sigma \underset{\sim}{M}_A = (7\underset{\sim}{k}) \times (B_x\underset{\sim}{i} + B_y\underset{\sim}{j} + B_z\underset{\sim}{k}) + (3\underset{\sim}{j} + 3.5\underset{\sim}{k}) \times (-mg\underset{\sim}{i}) + (6\underset{\sim}{j} + 2\underset{\sim}{k}) \times F_{CD}\,\underset{\sim}{n}_{CD} = \underset{\sim}{0}$$

③

$$7B_x\underset{\sim}{j} - 7B_y\underset{\sim}{i} + 3mg\underset{\sim}{k} - 3.5mg\underset{\sim}{j}$$

$$+ \frac{F_{CD}}{7}(-36\underset{\sim}{k} + 12\underset{\sim}{i} + 12\underset{\sim}{j} - 6\underset{\sim}{i}) = 0$$

$$\left(\frac{6}{7}F_{CD} - 7B_y\right)\underset{\sim}{i} + \left(7B_x + \frac{12}{7}F_{CD} - 3.5mg\right)\underset{\sim}{j}$$

$$+ \left(3mg - \frac{36}{7}F_{CD}\right)\underset{\sim}{k} = 0$$

$$\therefore F_{CD} = \frac{7}{12}mg = \frac{700}{12}g(N) = \frac{175}{3}g(N)$$

① $$\therefore B_y = \frac{6}{49}F_{CD} = \frac{50}{7}g(N)$$

① $$\therefore B_x = -\frac{12}{49}F_{CD} + 50g = \left(-\frac{100}{7} + 50\right)g = \frac{250}{7}g(N)$$

① $$\therefore A_x = mg - B_x - \frac{6}{7}F_{CD}$$

$$= 100g - \frac{250}{7}g - \frac{6}{7}\times\frac{700}{12}g$$

$$= 50g - \frac{250}{7}g = \frac{100}{7}g$$

① $$\therefore A_y = -B_y - \frac{3}{7}F_{CD}$$

$$= -\frac{100}{14}g - \frac{3}{7}\times\frac{700}{12}g = \left(-\frac{50}{7} - 25\right)g$$

$$\therefore A_n = \sqrt{A_x^2 + A_y^2}\ (①), \quad B_n = \sqrt{B_x^2 + B_y^2}\ (①)$$