Statics: Mid-Term Exam

2019. 10. 23. 11:00~13:00, Closed Book
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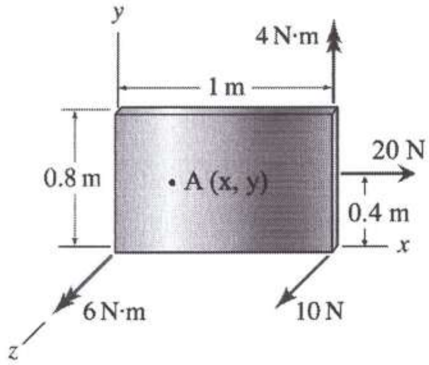
1. (20점) 그림에 주어진 힘에 대한 Wrench 등가힘계(equivalent force system)를 구하시오. 또한 Wrench 등가힘계의 작용선(line of action)이 xy 평면과 교차하는 점 A의 x 및 y 좌표를 구하시오.



$\underset{\sim}{R} = 20\underset{\sim}{i} + 10\underset{\sim}{k}$ (N)

$\underset{\sim}{M}_A = \Sigma \underset{\sim}{M}_i + \Sigma \underset{\sim}{r}_i \times \underset{\sim}{F}_i$

$\underset{\sim}{r}_1 = (1-x)\underset{\sim}{i} - y\underset{\sim}{j}$ (m), $\underset{\sim}{F}_1 = 10\underset{\sim}{k}$ (N)

$\underset{\sim}{r}_2 = (1-x)\underset{\sim}{i} + (0.4-y)\underset{\sim}{j}$ (m), $\underset{\sim}{F}_2 = 20\underset{\sim}{i}$ (N)

$\underset{\sim}{r}_1 \times \underset{\sim}{F}_1 = -10(1-x)\underset{\sim}{j} - 10y\underset{\sim}{i}$

$\underset{\sim}{r}_2 \times \underset{\sim}{F}_2 = -20(0.4-y)\underset{\sim}{k}$

$\therefore \underset{\sim}{M}_A = 4\underset{\sim}{j} + 6\underset{\sim}{k} - 10y\underset{\sim}{i} - 10(1-x)\underset{\sim}{j} - 20(0.4-y)\underset{\sim}{k}$

$= -10y\underset{\sim}{i} - (6-10x)\underset{\sim}{j} - (2-20y)\underset{\sim}{k}$

For wrench, $\underset{\sim}{R} \parallel \underset{\sim}{M}_A$

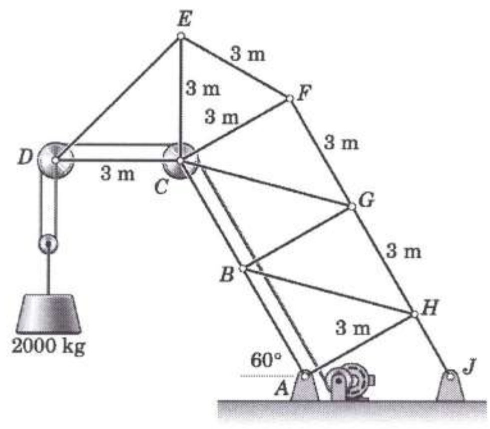
$\therefore 6-10x = 0 \rightarrow \therefore \underline{x = 0.6 \text{ m}}$

$\therefore \dfrac{20}{10y} = \dfrac{10}{2-20y} \rightarrow 4-40y = 10y$

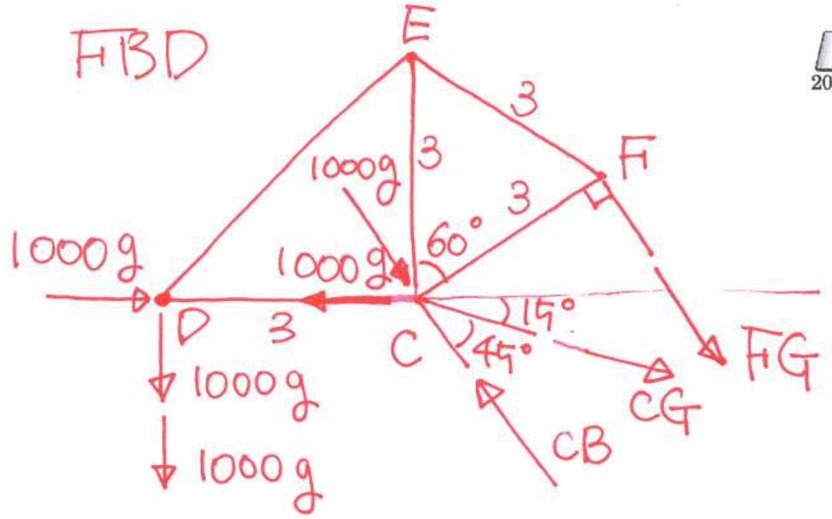
$\therefore \underline{y = 0.08 \text{ m}}$ & $\underline{x = 0.6 \text{ m}}$

$\therefore \underline{\underset{\sim}{R} = 20\underset{\sim}{i} + 10\underset{\sim}{k} \text{ (N)}}$ $\therefore \underline{\underset{\sim}{M}_A = -0.8\underset{\sim}{i} - 0.4\underset{\sim}{k} \text{ (N·m)}}$

2. (20점) 그림에 주어진 Truss 구조물에 대하여 부재 *CB*, *CG*, 및 *FG*에 걸리는 힘을 구하고, 압축인지 인장인지 기술하시오. (단, 중력가속도는 *g*이고, truss와 도르래의 자중은 무시한다.)



FBD



$$\Sigma M_c = 3 \times 2000g - 3FG = 0$$

$$\therefore FG = 2000g \ (N) \quad (\text{Tension})$$

$$\Sigma F_x = 1000g\cos 60^\circ + FG\cos 60^\circ - CB\cos 45^\circ + CG\cos 15^\circ = 0$$

$$\therefore CB\cos 45^\circ - CG\cos 15^\circ = 1500g \quad — ①$$

$$\Sigma F_y = CB\sin 45^\circ - CG\sin 15^\circ - 2000g - 1000g\sin 60^\circ - FG\sin 60^\circ = 0$$

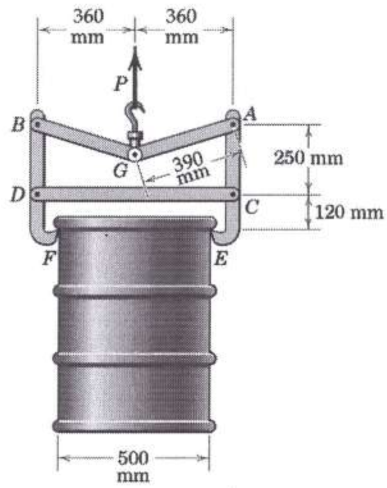
$$\therefore CB\sin 45^\circ - CG\sin 15^\circ = 2000g + 1500\sqrt{3}g \quad — ②$$

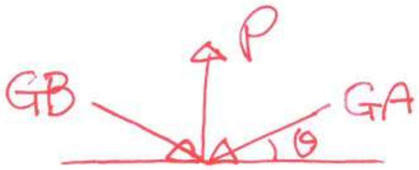
① 과 ② 를 연립하여 풀면

$$\therefore CG = 500g \times \frac{1+3\sqrt{3}}{\cos 15^\circ - \sin 15^\circ} \quad (N) \ (T)$$

$$\therefore CB = 500\sqrt{2}g \times \frac{(1+3\sqrt{3})\cos 15^\circ}{\cos 15^\circ - \sin 15^\circ} + 1500\sqrt{2}g \quad (N) \ (C)$$

3. (20점) 그림과 같은 150 kg의 steel drum을 운반하는 lift가 있다. E 점에서 steel drum에 작용하는 힘을 vector로 나타내라. 단, 중력가속도는 g이다.

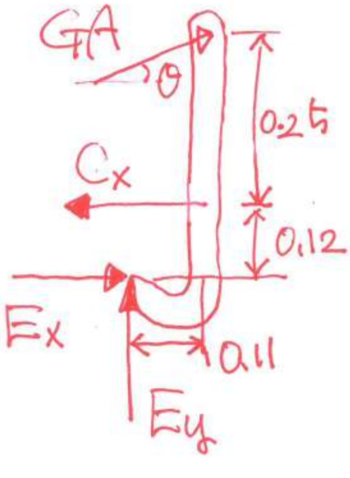




$$\cos\theta = \frac{12}{13}, \quad \sin\theta = \frac{5}{13}$$

$$\Sigma F_y = P - 2GA\sin\theta = 0$$

$$GA = \frac{P}{2\sin\theta} = 1.3 \times 150g = 195g \ (N)$$



$$\Sigma M_E = -GA\cos\theta \times 0.37 + GA\sin\theta \times 0.11 + C_x \times 0.12 = 0$$

$$\therefore C_x = 486.25\,g \ (N)$$

$$\Sigma F_x = E_x - C_x + GA\cos\theta = 0$$

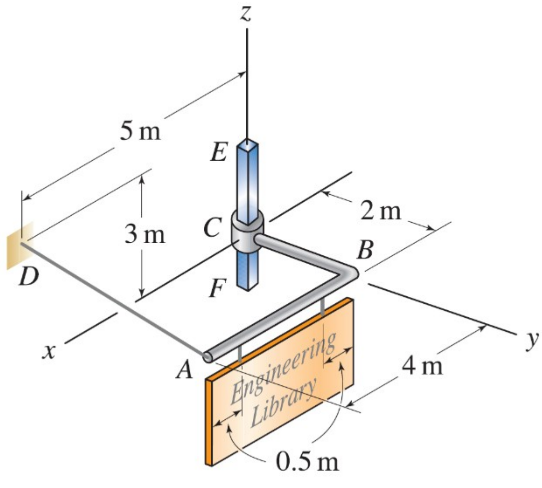
$$\therefore E_x = 306.25\,g \ (N)$$

$$\Sigma F_y = E_y + GA\sin\theta = 0$$

$$\therefore E_y = 75\,g \ (N)$$

$$\therefore \underset{\sim}{E} = -306.25\,g\,\underset{\sim}{i} - 75\,g\,\underset{\sim}{j} \ (N)$$

4. (20점) 그림과 같이 봉 EF는 정사각형 단면을 가지고 있으며 공간상에 고정되어 있다. 구조물 ABC의 무게는 무시할 수 있으며, 점 C에 있는 고리는 봉 EF 위에서 자유롭게 미끄러질 수 있다. 구조물 ABC는 무게가 1 kN인 균일한 직사각형의 간판을 지지하고 있다. Cable AD에 걸리는 장력의 크기를 구하고, 점 C에서 봉 EF가 구조물 ABC에 가하는 반력 (reaction force) 및 Moment를 vector로 나타내어라.



The vector expression for the force the cable applies to point A is

$$\vec{T}_{AD} = T_{AD}\frac{1\,\hat{\imath} - 2\,\hat{\jmath} + 3\,\hat{k}}{\sqrt{14}}. \qquad (1)$$

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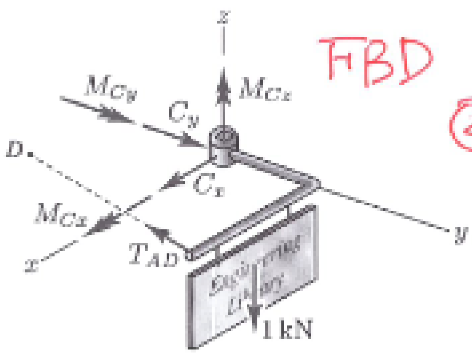


FBD

2

Equilibrium of forces requires

$$\sum \vec{F} = \vec{0}: \quad -1\,\text{kN}\,\hat{k} + \vec{T}_{AD} + C_x\,\hat{\imath} + C_y\,\hat{\jmath} = \vec{0}. \qquad (2)$$

2

Upon substitution and grouping of the components, Eq. (2) becomes

$$\left[C_x + \frac{1}{\sqrt{14}}T_{AD}\right]\hat{\imath} + \left[C_y - \frac{2}{\sqrt{14}}T_{AD}\right]\hat{\jmath} + \left[-1\,\text{kN} + \frac{3}{\sqrt{14}}T_{AD}\right]\hat{k} = \vec{0}. \qquad (3)$$

For Eq. (3) to be satisfied, each of the terms multiplying $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ must be zero, hence

$$C_x + \frac{1}{\sqrt{14}}T_{AD} = 0, \qquad (4)$$

$$C_y - \frac{2}{\sqrt{14}}T_{AD} = 0, \qquad (5)$$

$$-1\,\text{kN} + \frac{3}{\sqrt{14}}T_{AD} = 0. \qquad (6)$$

Solving Eq. (6) to obtain T_{AD}, and then using the other two equations to obtain C_x and C_y results in

2 $$\boxed{C_x = -\frac{1}{3}\,\text{kN} = -0.333\,\text{kN},} \qquad (7)$$

2 $$\boxed{C_y = \frac{2}{3}\,\text{kN} = 0.667\,\text{kN},} \qquad (8)$$

2 $$\boxed{T_{AD} = \frac{\sqrt{14}}{3}\,\text{kN} = 1.25\,\text{kN}.} \qquad (9)$$

To determine the moment reactions, we sum moments about a convenient point, choosing point C, to write

$$\sum \vec{M}_C = \vec{0}: \quad -1\,\text{kN}(2\,\text{m})\,\hat{\imath} + 1\,\text{kN}(2\,\text{m})\,\hat{\jmath} + M_{Cx}\,\hat{\imath} + M_{Cy}\,\hat{\jmath} + M_{Cz}\,\hat{k} + \vec{r}_{CA} \times \vec{T}_{AD} = \vec{0} \tag{10}$$ 2

$$-2\,\text{kN·m}\,\hat{\imath} + 2\,\text{kN·m}\,\hat{\jmath} + M_{Cx}\,\hat{\imath} + M_{Cy}\,\hat{\jmath} + M_{Cz}\,\hat{k} + \frac{T_{AD}}{\sqrt{14}} \det \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4\,\text{m} & 2\,\text{m} & 0 \\ 1 & -2 & 3 \end{bmatrix} = \vec{0} \tag{11}$$

$$\begin{aligned} -2\,\text{kN·m}\,\hat{\imath} + 2\,\text{kN·m}\,\hat{\jmath} + M_{Cx}\,\hat{\imath} + M_{Cy}\,\hat{\jmath} + M_{Cz}\,\hat{k} \\ + \frac{T_{AD}}{\sqrt{14}}\Big[(6\,\text{m})\,\hat{\imath} - (12\,\text{m})\,\hat{\jmath} + (-8\,\text{m} - 2\,\text{m})\,\hat{k}\Big] = \vec{0}. \end{aligned} \tag{12}$$

Grouping terms in Eq. (12) provides

$$\Big[-2\,\text{kN·m} + M_{Cx} + \frac{6\,\text{m}}{\sqrt{14}} T_{AD}\Big]\hat{\imath} + \Big[2\,\text{kN·m} + M_{Cy} - \frac{12\,\text{m}}{\sqrt{14}} T_{AD}\Big]\hat{\jmath} + \Big[M_{Cz} - \frac{10\,\text{m}}{\sqrt{14}} T_{AD}\Big]\hat{k} = \vec{0}, \tag{13}$$

which can be expressed as the three scalar equations

$$-2\,\text{kN·m} + M_{Cx} + \frac{6\,\text{m}}{\sqrt{14}} T_{AD} = 0, \tag{14}$$

$$2\,\text{kN·m} + M_{Cy} - \frac{12\,\text{m}}{\sqrt{14}} T_{AD} = 0, \tag{15}$$

$$M_{Cz} - \frac{10\,\text{m}}{\sqrt{14}} T_{AD} = 0. \tag{16}$$

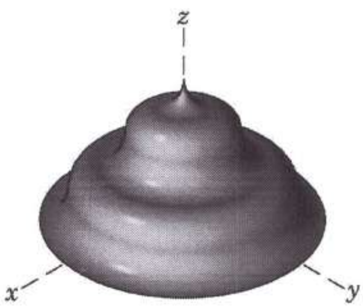
With T_{AD} already known, the above equations are easily solved to obtain

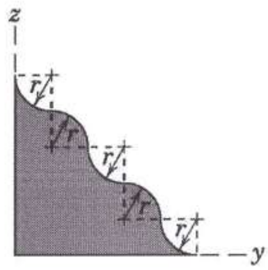
2 $$\boxed{M_{Cx} = 0,} \tag{17}$$

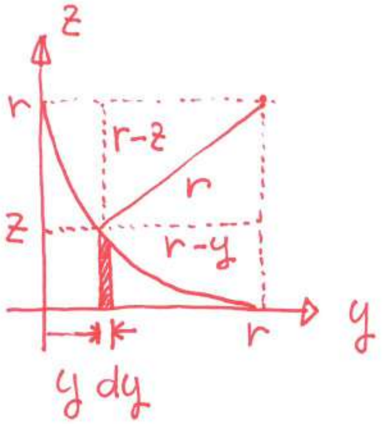
2 $$\boxed{M_{Cy} = 2\,\text{kN·m},} \tag{18}$$

2 $$\boxed{M_{Cz} = \frac{10}{3}\,\text{kN·m} = 3.\dot{3}3\,\text{kN·m}.} \tag{19}$$

5. (20점) 그림과 같이 오른쪽에 주어진 단면을 z 축에 대해 360° 회전하여 얻어진 황동 장식물의 무게를 계산하라. 황동의 밀도는 ρ이고, 중력가속도는 g이다.







$$A = r^2\left(1-\frac{\pi}{4}\right) \quad ①$$

$$r^2 = (r-y)^2 + (r-z)^2$$

$$z = r - \sqrt{r^2-(r-y)^2}$$

$$A\bar{y}_1 = \int_0^r y\,dA = \int_0^r y\,(z\,dy) = \int_0^r y\left[r-\sqrt{r^2-(r-y)^2}\right]dy$$

Let $r-y = r\cos\theta$, $y = r(1-\cos\theta)$, & $dy = r\sin\theta\,d\theta$

$$A\bar{y}_1 = \int_0^{\frac{\pi}{2}} r(1-\cos\theta)\,r(1-\sin\theta)\,r\sin\theta\,d\theta$$

$$= r^3\int_0^{\frac{\pi}{2}}\left(\sin\theta - \sin\theta\cos\theta - \sin^2\theta + \sin^2\theta\cos\theta\right)d\theta$$

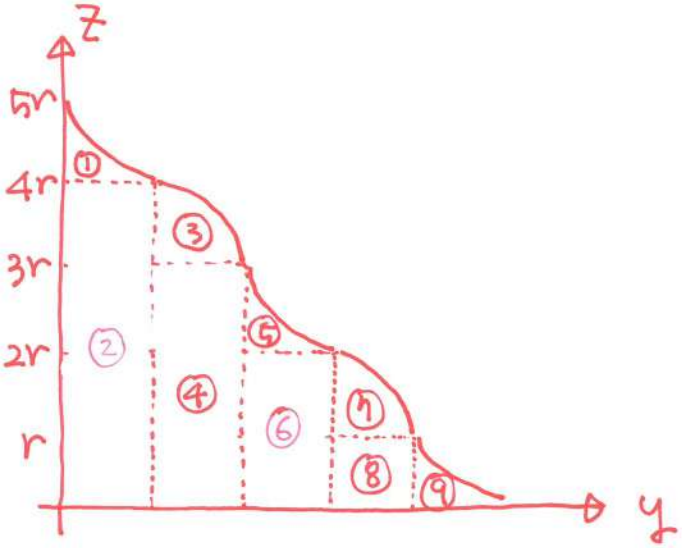
$$= r^3\int_0^{\frac{\pi}{2}}\left(\sin\theta - \frac{1}{2}\sin 2\theta - \frac{1-\cos 2\theta}{2} + \sin^2\theta\cos\theta\right)d\theta$$

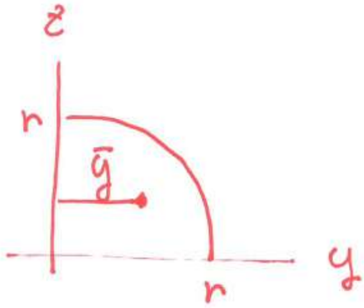
$$= r^3\left[-\cos\theta + \frac{1}{4}\cos 2\theta - \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + \frac{1}{3}\sin^3\theta\right]_0^{\frac{\pi}{2}}$$

$$= r^3\left(1-\frac{1}{2}-\frac{\pi}{4}+\frac{1}{3}\right)$$

$$= \left(\frac{5}{6}-\frac{\pi}{4}\right)r^3$$

$$\therefore\ \bar{y}_1 = \left(\frac{10/3-\pi}{4-\pi}\right)r. \quad ⑤$$





$A = \frac{\pi}{4} r^2$ ①

$\bar{y} = \frac{4r}{3\pi}$ ③

$$V = 2\pi \sum \bar{y}_i A_i$$

$$\frac{V}{2\pi} = \left(\frac{5}{6} - \frac{\pi}{4}\right) r^3 + \frac{r}{2} \times 4r^2$$

$$+ \left(1 + \frac{4}{3\pi}\right) r \times \frac{\pi}{4} r^2 + \frac{3}{2} r \times 3r^2$$

$$+ \left(2r + \frac{\frac{10}{3} - \pi}{4 - \pi} r\right) \times \left(1 - \frac{\pi}{4}\right) r^2 + \frac{5}{2} r \times 2r^2$$

$$+ \left(3 + \frac{4}{3\pi}\right) r \times \frac{\pi}{4} r^2 + \frac{7}{2} r \times r^2$$

$$+ \left(4r + \frac{\frac{10}{3} - \pi}{4 - \pi} r\right) \times \left(1 - \frac{\pi}{4}\right) r^2$$

$$= \left(24 + \frac{1}{6} - \frac{5}{4}\pi\right) r^3$$

$$\therefore V = \left(48 + \frac{1}{3} - \frac{5}{2}\pi\right) \pi r^3 \quad ⑥$$

$$\therefore \underbrace{W = \rho g V}_{①} = \underbrace{\left(48 + \frac{1}{3} - \frac{5}{2}\pi\right) \pi \rho g r^3}_{③}$$