$\cancel{\frac{z^4}{b^4}} - \frac{2z^2}{b^2} + 1 - \cancel{\frac{z^6}{b^6}} + \cancel{\frac{2z^4}{b^4}} - \frac{z^2}{b^2}$

$-\frac{z^6}{b^6} + \frac{3z^4}{b^4} - \frac{3z^2}{b^2} + 1$

Statics: Final Exam

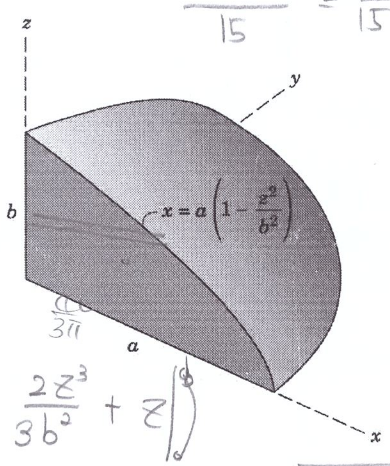
2018. 12. 19. 11:00~13:00, Closed Book
Instructor: Seungtae CHOI

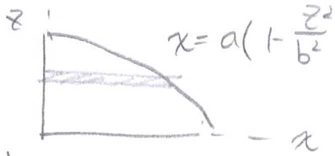
Student ID: 20180082 Name: 박민우 Seat No: B1

$\frac{3-10+15}{15} = \frac{8}{15} \times \frac{1}{4}$

20

1. (20 points) Determine the coordinates of the mass center of the solid homogeneous body formed by revolving the shaded area 90° about the z-axis.





$dV = \frac{\pi}{4} x^2 dz$ 2

2

$V = \int dV = \frac{\pi}{4}\int_0^b a^2\left(1-\frac{z^2}{b^2}\right)^2 dz$ 1

$= \frac{\pi a^2}{4}\int_0^b \left(\frac{z^4}{b^4} - \frac{2z^2}{b^2} + 1\right) dz = \frac{\pi a^2}{4} \times \left(\frac{1}{5b^4}z^5 - \frac{2z^3}{3b^2} + z\right)\Big|_0^b$

$= \frac{\pi a^2}{4}\left(\frac{b^5}{5b^4} - \frac{2b^3}{3b^2} + b\right) = \frac{\pi a^2 b}{4}\left(\frac{1}{5} - \frac{2}{3} + 1\right) = \frac{2\pi a^2 b}{15}$ 5

$x_c = \frac{4x}{3\pi}$
$y_c = \frac{4y}{3\pi}$
$z_c = z$

$V\bar{x} = \int x_c dV = \frac{\pi}{4}\int_0^b \frac{4x}{3\pi} a^2\left(1-\frac{z^2}{b^2}\right)^2 dz$ b^4

$= \frac{1}{3}\int_0^b a^3\left(1-\frac{z^2}{b^2}\right)\left(\frac{z^4}{b^4} - \frac{2z^2}{b^2} + 1\right) dz$

$= \frac{a^3}{3}\int_0^b \left(-\frac{z^6}{b^6} + \frac{3z^4}{b^4} - \frac{3z^2}{b^2} + 1\right) dz$ $\frac{-5+21}{35}$ $\frac{16}{35} \times \frac{1}{3}$

$= \frac{a^3}{3} \times \left[-\frac{z^7}{7b^6} + \frac{3z^5}{5b^4} - \frac{z^3}{b^2} + z\right]_0^b = \frac{a^3 b}{3} \times \left(-\frac{1}{7} + \frac{3}{5} - 1 + 1\right)$

$= \frac{16a^3 b}{105}$

(i) $\bar{x} = \frac{16a^3 b}{105} \times \frac{15}{2\pi a^2 b} = \frac{16a}{14\pi} \to \frac{8}{7\pi}a$ 5

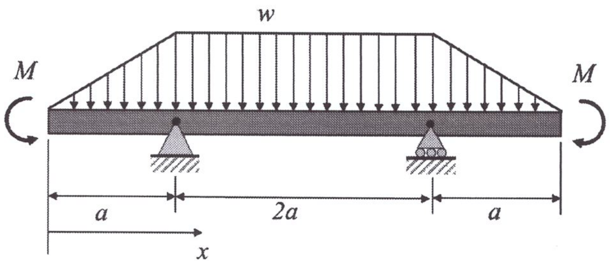
(ii) $\bar{y}$ = Same as $\bar{x}$

$V\bar{z} = \int z\, dV = \frac{\pi a^2}{4}\int_0^b z\left(1-\frac{z^2}{b^2}\right)^2 dz = \frac{\pi a^2}{4}\int_0^b \left(\frac{z^5}{b^4} - \frac{2z^3}{b^2} + z\right) dz$

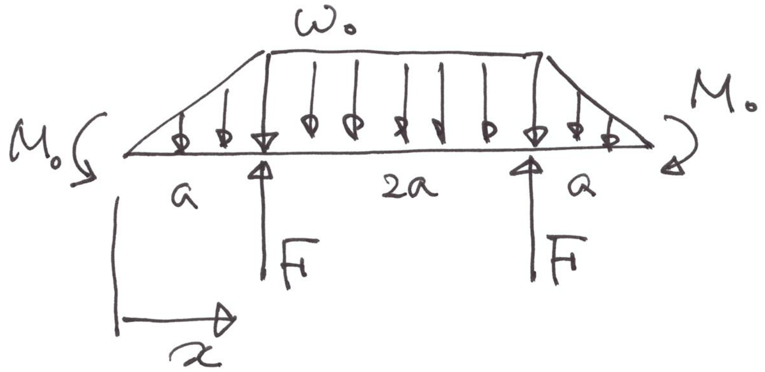
$= \frac{\pi a^2}{4}\left[\frac{z^6}{6b^4} - \frac{z^4}{2b^2} + \frac{1}{2}z^2\right]_0^b = \frac{\pi a^2 b^2}{4}\left(\frac{1}{6} - \frac{1}{2} + \frac{1}{2}\right) = \frac{\pi a^2 b^2}{24}$

$\bar{z} = \frac{\pi a^2 b^2}{24} \times \frac{15}{2\pi a^2 b} = \frac{5}{16}b$ 5

2. (20 points)
   (a) Draw the shear force and bending moment diagrams for the beam. (Please use $M = wa^2$)
   (b) Find out the moments at $x = a$, $2a$, and $3a$.

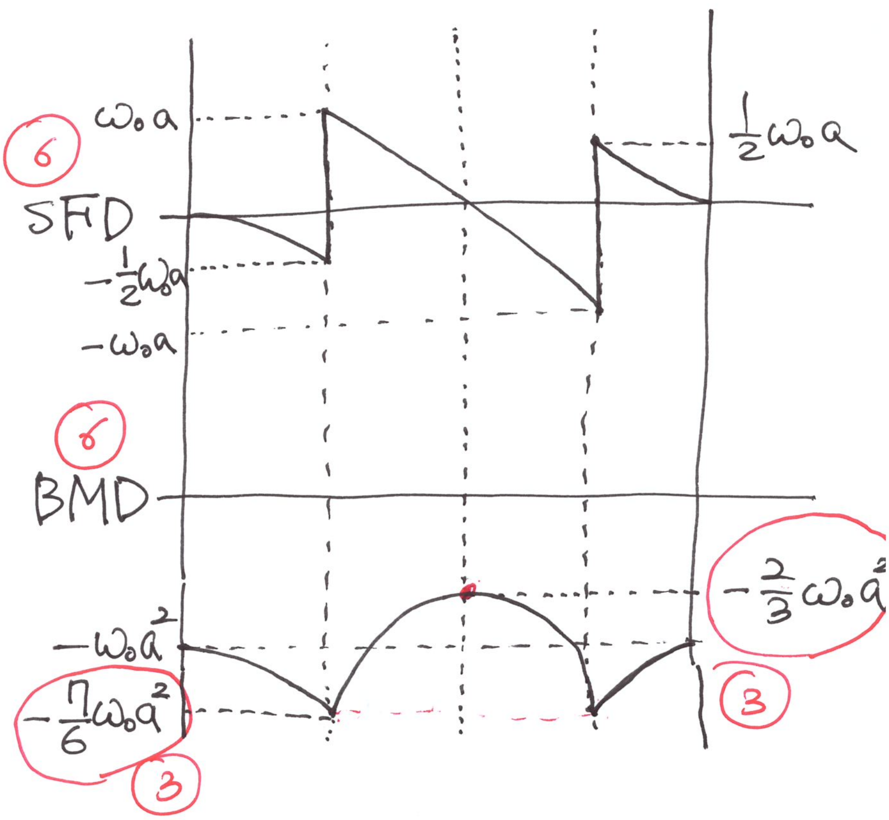


Free Body Diagram



$$\Sigma F_y = 0$$

$$\rightarrow F = \frac{3}{2}\omega_o a \quad 2$$



i) $0 < x < a$

$$\omega = \frac{\omega_o}{a} x$$

$$\frac{dV}{dx} = -\omega = -\frac{\omega_o}{a} x$$

$$\therefore V = -\frac{\omega_o}{2a} x^2$$

$$\therefore M = -\frac{\omega_o}{6a} x^3 - M_o$$

$$\left(\because \frac{dM}{dx} = V\right)$$

ii) $a < x < 3a$

$$\omega = \omega_o$$

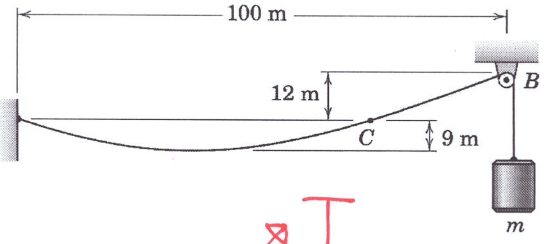
$$V = -\omega_o x + V_1 = -\omega_o x + 2\omega_o a$$

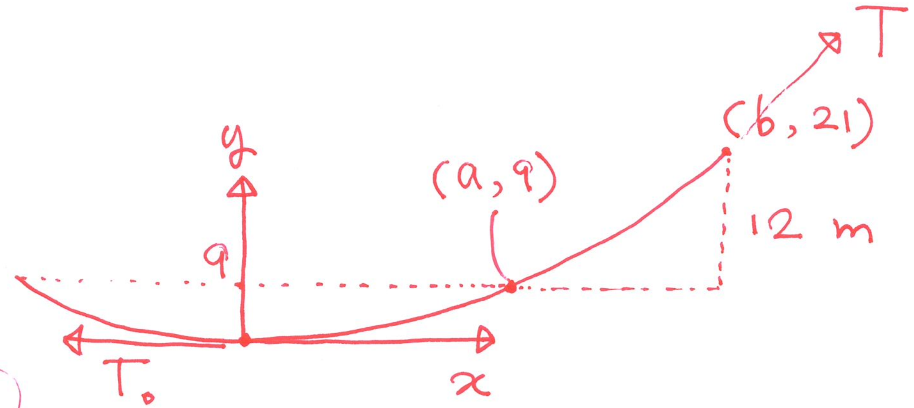
$$M = -\frac{1}{2}\omega_o x^2 + 2\omega_o a x + M_1$$

$$M_1 = -\frac{16}{6}\omega_o a^2$$

iii) $3a < x < 4a$: Apply the symmetry!!

3. (20 points) A cable weighing 10 N per meter of length is suspended from point A and passes over the small pulley at B. Because of the small sag-to-span ratio, use the approximation of a parabolic cable.
   (a) Calculate the mass m of the attached cylinder which will produce the sag of 9 m.
   (b) Determine the horizontal distance from A to C.





2

$a+b = 100$ m, $\omega = 10$ N/m

Governing Eq. $\frac{d^2y}{dx^2} = \frac{\omega}{T_0}$

$y = \frac{\omega x^2}{2T_0}$ 3

$9 = \frac{10}{2T_0} a^2 \rightarrow a = \sqrt{1.8T_0}$

$21 = \frac{10}{2T_0} b^2 \rightarrow b = \sqrt{4.2T_0} = \sqrt{\frac{7}{3}}\, a$

$a+b = \left(1+\sqrt{\frac{7}{3}}\right)a = 100$

$\therefore a = \cancel{100}\ 100 \Big/ \left(1+\sqrt{\frac{7}{3}}\right)$ $\quad \therefore \overline{AC} = 2a = \frac{200}{1+\sqrt{\frac{7}{3}}}$

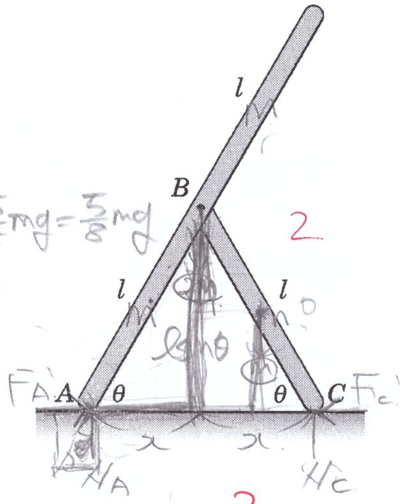
$\therefore T_0 = \frac{a^2}{1.8} = \frac{100^2}{1.8\left(1+\sqrt{\frac{7}{3}}\right)^2}$

$T^2 = T_0^2 + (\omega b)^2$ 10

$\therefore m = \frac{T}{g} = \frac{1}{g}\sqrt{T_0^2 + \frac{7}{3}\omega^2 a^2}$ 5

20

4. (20 points) The two uniform slender bars constructed from the same material are freely pinned together at B. The coefficient of static friction at both A and C is $\mu_s = 0.50$. Consider only motion in the vertical plane shown. Determine the minimum angle θ at which slipping does not occur at either contact point A or C.



2

2

길이 l당 질량을 m이라고 설정

$\Sigma M_A = 0$

3

$2mg \times x + mg \times \frac{3}{2}x = 2x \times N_C$

$2N_C = (2m + \frac{3}{2}m)g$

$= \frac{7}{2}mg$

$\therefore N_C = \frac{7}{4}mg$

(전체) $\Sigma F_y = 0$

$2mg + mg = N_A + N_C$

$N_A = 3mg - \frac{7}{4}mg$

$= \frac{5}{4}mg$

ㄴ) 최소각 θ

⇒) 미끄러지지 않는

$F_A = 0.50 \times N_A$ $= \frac{1}{2} \times \frac{5}{4}mg = \frac{5}{8}mg$

4

$F_C = 0.50 \times N_C$

$= 0.5 \times \frac{7}{4}mg$

$= \frac{7}{8}mg$

4

$\tan\theta = \frac{N_A}{F_A} = \frac{N_A}{0.5 N_A} = 2$

$\boxed{\theta = \tan^{-1}(2)}$

2

$\Sigma M_B = 0$ 힘에

3

↻ N_A F_C $l \cdot mg$ ↺ N_C F_A

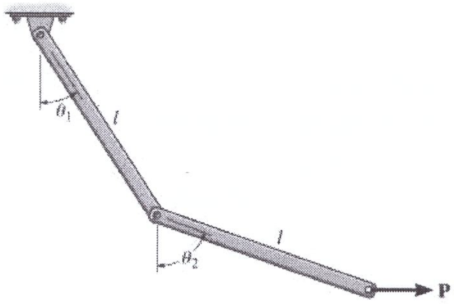
$\frac{5}{4}mgx + \frac{7}{8}mg\,l\sin\theta + mg\frac{x}{2} = \frac{7}{4}mgx + \frac{5}{8}mg\,l\sin\theta$

$\frac{5}{4}x + \frac{7}{8}l\sin\theta + \frac{x}{2} = \frac{7}{4}x + \frac{5}{8}l\sin\theta$

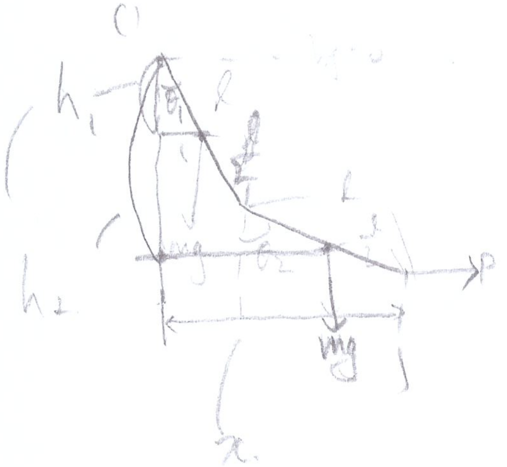
$\frac{2}{8}l\sin\theta = \frac{14-10-4}{8}$

20

5. (20 points) A horizontal force acts on the end of the link as shown. Determine the angles θ_1 and θ_2 for equilibrium for the two links. Each link is uniform and has a mass m.



Active force diagram을 그리면



$\delta W' = 0$ 이어야 한다.

$\delta W' = +mg\,\delta h_1 + mg\,\delta h_2 + P\,\delta x = 0$

이때 $h_1 = \frac{l}{2}\cos\theta_1 \Rightarrow \delta h_1 = -\frac{l}{2}\sin\theta_1\,\delta\theta_1$ 3

$h_2 = l\cos\theta_1 + \frac{l}{2}\cos\theta_2 \Rightarrow \delta h_2 = -l\sin\theta_1\,\delta\theta_1 - \frac{l}{2}\sin\theta_2\,\delta\theta_2$ 3

$x = l\sin\theta_1 + l\sin\theta_2 \Rightarrow \delta x = l\cos\theta_1\,\delta\theta_1 + l\cos\theta_2\,\delta\theta_2$

$Pl(\cos\theta_1\,\delta\theta_1 + \cos\theta_2\,\delta\theta_2) = mgl\left(\frac{\sin\theta_1}{2}\,\delta\theta_1 + \sin\theta_1\,\delta\theta_1 + \frac{1}{2}\sin\theta_2\,\delta\theta_2\right)$ 4

$\Rightarrow \delta\theta_1 \times \Big(\underbrace{P\cos\theta_1 - \frac{mg\sin\theta_1}{2} - mg\sin\theta_1}_{0 \text{ 이어야 함}}\Big) + \delta\theta_2\Big(\underbrace{P\cos\theta_2 - \frac{mg}{2}\sin\theta_2}_{0 \text{ 이어야}}\Big) = 0$

$P\cos\theta_2 = \frac{mg}{2}\sin\theta_2 \quad \therefore \tan\theta_2 = \frac{2P}{mg} \quad \boxed{\theta_2 = \tan^{-1}\left(\frac{2P}{mg}\right)}$ 5

$P\cos\theta_1 = mg\left(\frac{3}{2}\sin\theta_1\right) \quad \therefore \tan\theta_1 = \frac{2P}{3mg} \quad \boxed{\theta_1 = \tan^{-1}\left(\frac{2P}{3mg}\right)}$

5