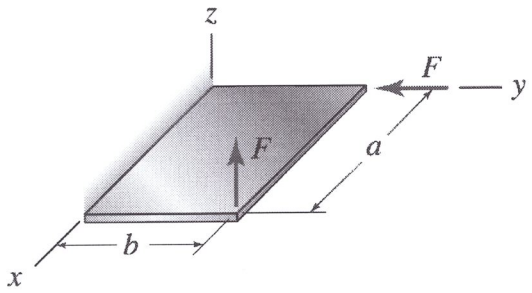
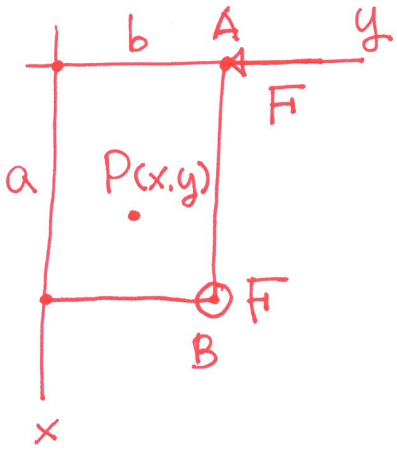
Statics: Mid-Term Exam

2018. 10. 24. 11:00~13:00, Closed Book
Instructor: Seungtae CHOI

Student ID:_____ Name:_____ Seat No:_____

1. (20점) 그림에 주어진 힘에 대한 Wrench 등가힘계(equivalent force system)를 구하시오. 또한 Wrench 등가힘계의 작용선(line of action)이 xy 평면과 교차하는 점의 x 및 y 좌표를 구하시오. (답을 F, a, 및 b로 표현 하시오.)





$$\Sigma \underset{\sim}{F} = -F\underset{\sim}{j} + F\underset{\sim}{k}$$

$$\Sigma \underset{\sim}{M}_P = \underset{\sim}{r}_{PA} \times (-F\underset{\sim}{j}) + \underset{\sim}{r}_{PB} \times (F\underset{\sim}{k})$$

$$= (-x\underset{\sim}{i} + (b-y)\underset{\sim}{j}) \times (-F\underset{\sim}{j}) + ((a-x)\underset{\sim}{i} + (b-y)\underset{\sim}{j}) \times F\underset{\sim}{k}$$

$$\Sigma \underset{\sim}{M}_P = xF\underset{\sim}{k} - (a-x)F\underset{\sim}{j} + (b-y)F\underset{\sim}{i}$$

In wrench equivalent force system,

$$\Sigma \underset{\sim}{F} \;//\; \Sigma \underset{\sim}{M}$$

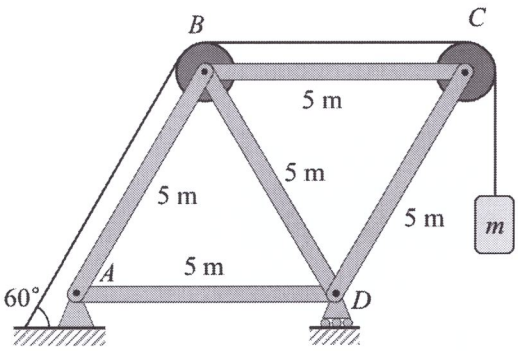
$$\therefore (b-y)F = 0 \rightarrow \therefore y = b$$

$$\frac{-(a-x)F}{-F} = \frac{xF}{F} \rightarrow \therefore x = \frac{a}{2}$$

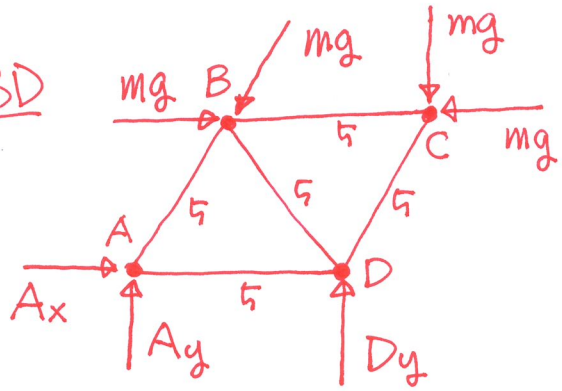
$$\therefore \underset{\sim}{F} = -F\underset{\sim}{j} + F\underset{\sim}{k}, \quad \underset{\sim}{M} = \frac{1}{2}aF(-\underset{\sim}{j} + \underset{\sim}{k})$$

$$\therefore x = \frac{a}{2}, \; y = b$$

2. (20점) 그림과 같이 길이가 모두 5 m로 같은 부재(member)들로 이루어진 Truss 구조물에 질량이 m인 물체가 도르래(pulley)와 줄(cable)로 매달려 있다. 이 때 부재 BD에 걸리는 힘을 구하고, 인장(tension)인지 압축(compression)인지 나타내시오. 단, 중력가속도는 g이다.



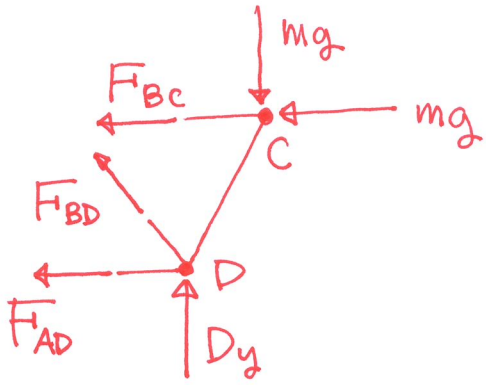
FBD



$$\Sigma M_A = 5D_y - 7.5mg = 0$$

$$\therefore D_y = 1.5mg$$

FBD

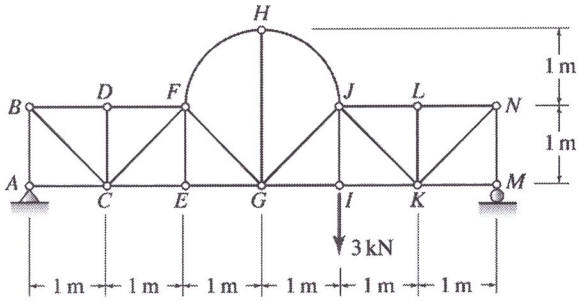


$$\Sigma F_y = D_y - mg + F_{BD}\sin 60^\circ = 0$$

$$F_{BD} = -\frac{0.5mg}{\sin 60^\circ}$$

$$\therefore F_{BD} = \frac{1}{\sqrt{3}}mg \quad \text{(Compression)}$$

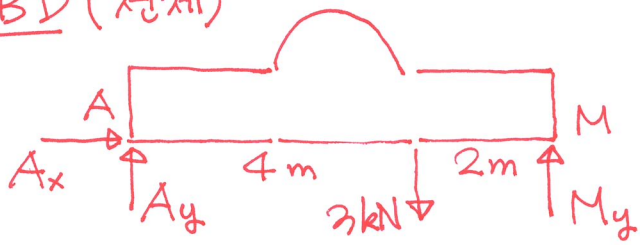
3. (20점) 아래에 주어진 Truss 구조물에 대하여 다음 질문에 답하시오.
   (1) 주어진 Truss에서 모든 무력 부재(zero-force member)를 나타내시오.
   (2) 점 G에 연결된 모든 부재에 걸리는 힘을 구하고, 인장(tension)인지 압축(compression)인지 나타내시오.



(1) Zero-force member:

CD, EF, KL

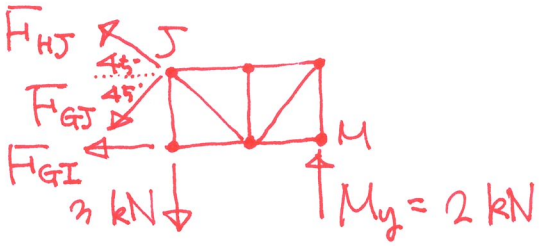
(2) FBD (전체)



$\Sigma M_A = 6M_y - 4\times 3 = 0$

$\therefore M_y = 2$ kN

FBD (단면)



$\Sigma M_J = 2M_y - 1\times F_{GI} = 0$

$\therefore F_{GI} = 4$ kN

$\Sigma F_x = -\frac{1}{\sqrt{2}}(F_{HJ} + F_{GJ}) - F_{GI} = 0$

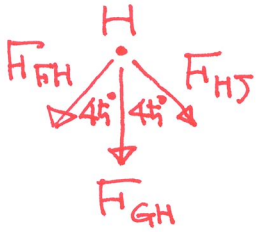
$\therefore F_{HJ} + F_{GJ} = -4\sqrt{2}$

$\Sigma F_y = \frac{1}{\sqrt{2}}(F_{HJ} - F_{GJ}) + 2 - 3 = 0$

$\therefore F_{HJ} - F_{GJ} = \sqrt{2}$

$\therefore F_{HJ} = -\frac{3}{2}\sqrt{2}$ kN, $F_{GJ} = -\frac{5}{2}\sqrt{2}$ kN

FBD (Joint H)

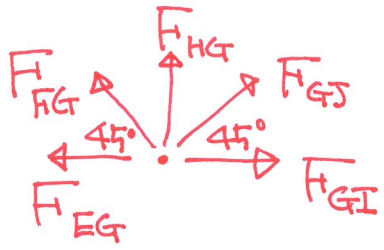


$\Sigma F_x = 0 \rightarrow F_{FH} = F_{HJ} = -\frac{3}{2}\sqrt{2}$ kN

$\Sigma F_y = 0 \rightarrow F_{GH} = -\sqrt{2}F_{HJ} = 3$ kN

Prob. 3 (Continued)

FBD (Joint G)



$$\Sigma F_y = F_{HG} + \frac{1}{\sqrt{2}}\left(F_{FG} + F_{GJ}\right) = 0$$

$$3 + \frac{1}{\sqrt{2}}\left(F_{FG} - \frac{5}{2}\sqrt{2}\right) = 0$$

$$\therefore F_{FG} = -\frac{1}{2}\sqrt{2}\ \text{kN}$$

$$\Sigma F_x = F_{GI} - F_{EG} + \frac{1}{\sqrt{2}}F_{GJ} - \frac{1}{\sqrt{2}}F_{FG} = 0$$

$$4 - F_{EG} - \frac{5}{2} + \frac{1}{2} = 0$$

$$\therefore F_{EG} = 2\text{kN}$$

$\therefore F_{EG} = 2\ \text{kN}\ (T)$

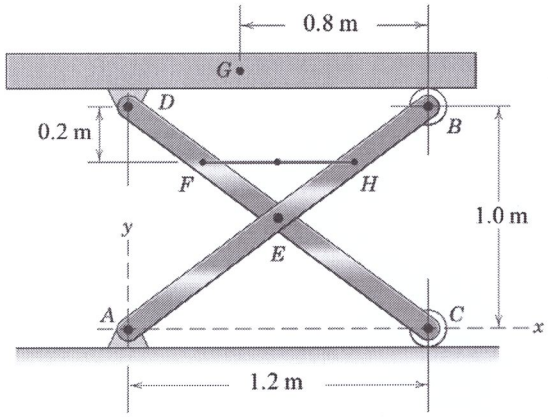
$\therefore F_{FG} = \frac{1}{\sqrt{2}}\ \text{kN}\ (C)$

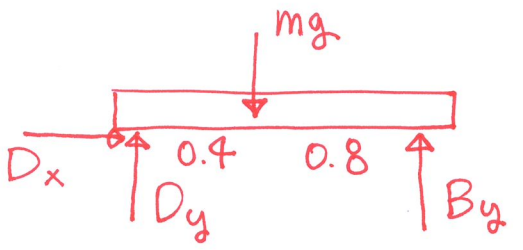
$\therefore F_{HG} = 3\ \text{kN}\ (T)$

$\therefore F_{GJ} = \frac{5}{2}\sqrt{2}\ \text{kN}\ (C)$

$\therefore F_{GI} = 4\ \text{kN}\ (T)$

4. (20점) 그림과 같은 접이식 작업대(folding workbench)의 상판(top plate)의 질량중심(mass center)은 G이고, 질량은 m이다. FH는 cable이다. 이때 frame $DFEC$가 점 E를 통해 frame $BHEA$에 가하는 힘을 vector로 나타내시오. 단, 중력가속도는 g이다.

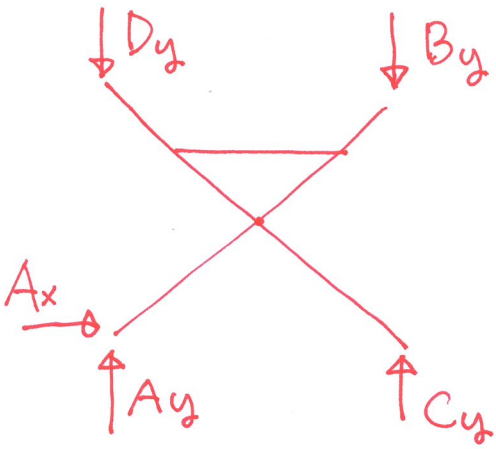




$\Sigma F_x = 0 \rightarrow D_x = 0$

$\Sigma M_D = 0 \rightarrow B_y = \frac{1}{3} mg$

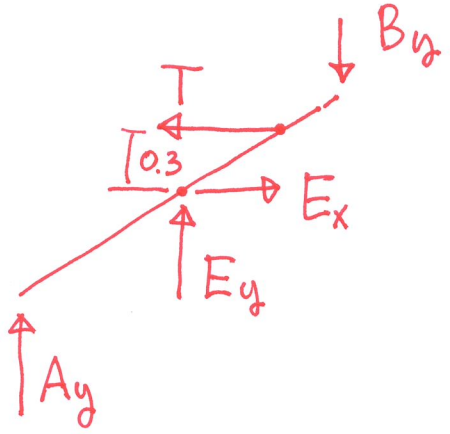
$\Sigma F_y = 0 \rightarrow D_y = \frac{2}{3} mg$



$\Sigma F_x = 0 \rightarrow A_x = 0$

$\Sigma M_A = 0 \rightarrow C_y = B_y = \frac{1}{3} mg$

$\Sigma F_y = 0 \rightarrow A_y = \frac{2}{3} mg$



$\Sigma M_E = 0.3T - 0.6(A_y + B_y) = 0$

$\therefore T = 2mg$

$\Sigma F_x = E_x - T = 0$

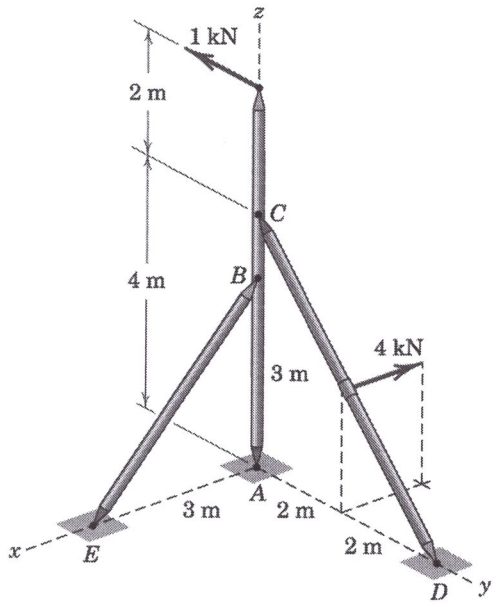
$\therefore E_x = 2mg$

$\Sigma F_y = A_y + E_y - B_y = 0$

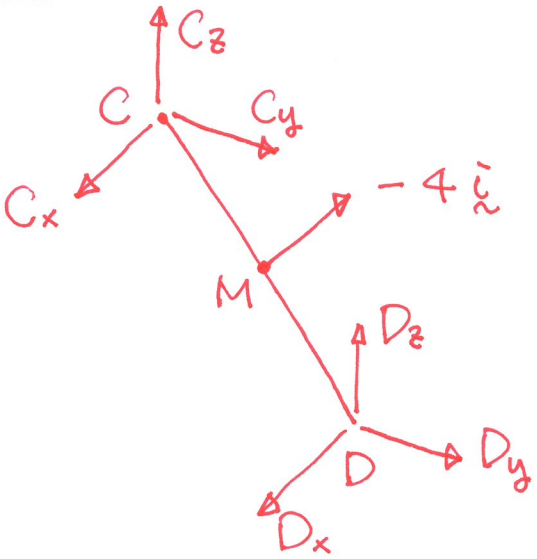
$\therefore E_y = -\frac{1}{3} mg$

$\therefore \underline{\vec{F}_E = \left(2\hat{i} - \frac{1}{3}\hat{j}\right) mg}$

5. (20점) 그림과 같은 구조물의 지지점 A에서 frame ABC에 작용하는 힘을 구하시오. 모든 연결부위는 ball-and-socket joint이며, 모든 구조물의 질량은 무시할 수 있다.



FBD of member CD



$$\Sigma \underset{\sim}{F} = \underset{\sim}{C} + \underset{\sim}{D} - 4\underset{\sim}{i} = \underset{\sim}{0}$$

$$\Sigma \underset{\sim}{M}_M = \underset{\sim}{r}_{MC} \times \underset{\sim}{C} + \underset{\sim}{r}_{MD} \times \underset{\sim}{D} = \underset{\sim}{0}$$

$$\underset{\sim}{r}_{MC} = -\underset{\sim}{r}_{MD}$$

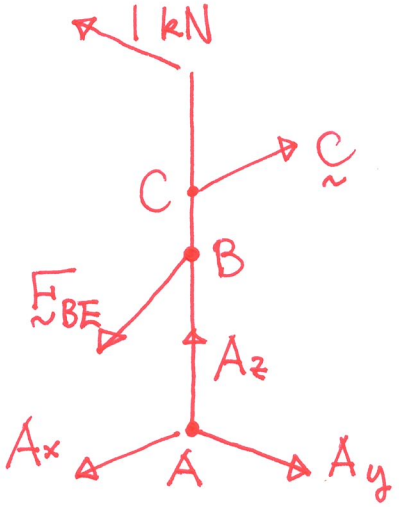
$$\underset{\sim}{r}_{MC} \times (\underset{\sim}{C} - \underset{\sim}{D}) = \underset{\sim}{0}$$

$$\underset{\sim}{r}_{MC} \times (2\underset{\sim}{C} - 4\underset{\sim}{i}) = \underset{\sim}{0}$$

$\therefore (2\underset{\sim}{C} - 4\underset{\sim}{i})$ is parallel to $\underset{\sim}{r}_{MC}$.

$$\underset{\sim}{C} = 2\underset{\sim}{i} + \alpha(-\underset{\sim}{j} + \underset{\sim}{k})$$

FBD of frame ABC



$$\underset{\sim}{F}_{BE} = F_{BE}\left(\frac{1}{\sqrt{2}}\underset{\sim}{i} - \frac{1}{\sqrt{2}}\underset{\sim}{k}\right)$$

$$\Sigma \underset{\sim}{M}_A = 6\underset{\sim}{k} \times (-\underset{\sim}{j}) + 4\underset{\sim}{k} \times (-2\underset{\sim}{i} + \alpha\underset{\sim}{j} - \alpha\underset{\sim}{k}) + 3\underset{\sim}{k} \times F_{BE}\left(\frac{1}{\sqrt{2}}\underset{\sim}{i} - \frac{1}{\sqrt{2}}\underset{\sim}{k}\right) = \underset{\sim}{0}$$

Prob. 5 (Continued)

$$6\underset{\sim}{i} - 8\underset{\sim}{j} - 4\alpha\underset{\sim}{i} + \frac{3}{\sqrt{2}}F_{BE}\underset{\sim}{j} = \underset{\sim}{0}$$

$$\therefore \alpha = \frac{3}{2}$$

$$\therefore F_{BE} = \frac{8}{3}\sqrt{2}$$

$$\therefore \underset{\sim}{C} = -2\underset{\sim}{i} + \frac{3}{2}\underset{\sim}{j} - \frac{3}{2}\underset{\sim}{k}$$

$$\therefore \underset{\sim}{F}_{BE} = \frac{8}{3}(\underset{\sim}{i} - \underset{\sim}{k})$$

$$\Sigma\underset{\sim}{F} = \underset{\sim}{A} + \underset{\sim}{C} + \underset{\sim}{F}_{BE} - \underset{\sim}{j} = \underset{\sim}{0}$$

$$\therefore \underset{\sim}{A} = -\underset{\sim}{C} - \underset{\sim}{F}_{BE} + \underset{\sim}{j}$$

$$= 2\underset{\sim}{i} - \frac{3}{2}\underset{\sim}{j} + \frac{3}{2}\underset{\sim}{k} - \frac{8}{3}\underset{\sim}{i} + \frac{8}{3}\underset{\sim}{k} + \underset{\sim}{j}$$

$$= -\frac{2}{3}\underset{\sim}{i} - \frac{1}{2}\underset{\sim}{j} + \frac{25}{6}\underset{\sim}{k}$$

$$\therefore \underline{\underset{\sim}{A} = -\frac{2}{3}\underset{\sim}{i} - \frac{1}{2}\underset{\sim}{j} + \frac{25}{6}\underset{\sim}{k} \quad (kN)}$$