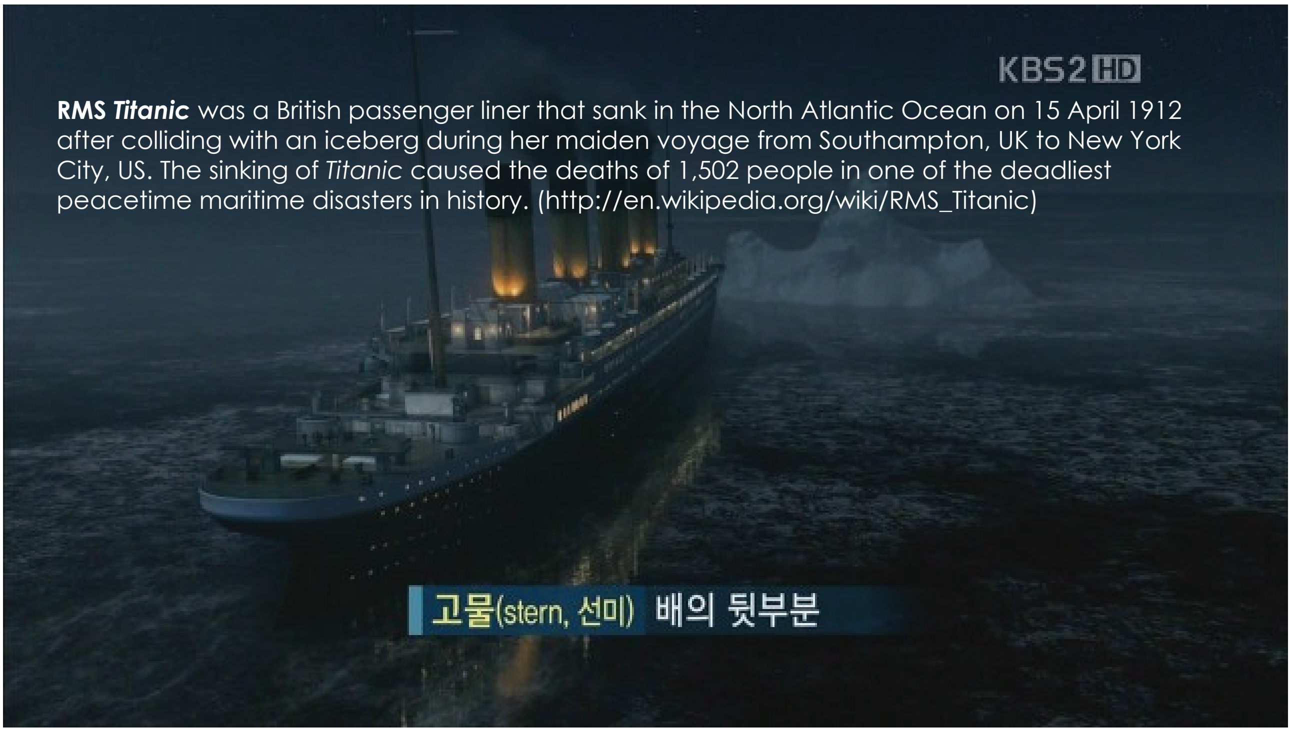
Chapter 10: Failure

- **ISSUES TO ADDRESS**
  - How do cracks that lead to failure form?
  - How is fracture resistance quantified? How do the fracture resistances of the different material classes compare?
  - How do we estimate the stress to fracture?
  - How do loading rate, loading history, and temperature affect the failure behavior of materials?

- **Failure Modes:**
  - Fracture
  - Fatigue
  - Creep

Titanic on April 15, 1912



10.3 Ductile Fracture (Rupture)

- **Classification of fracture behavior**

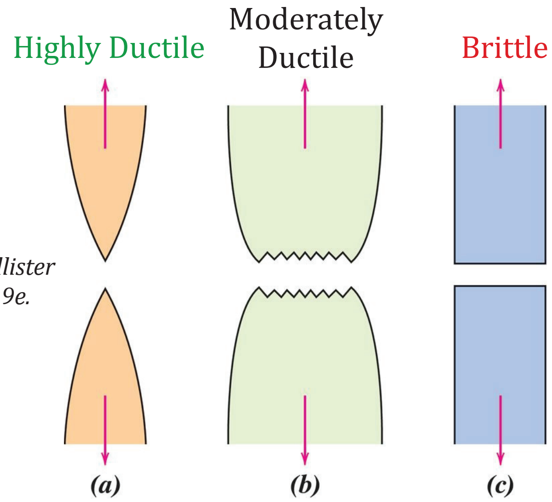
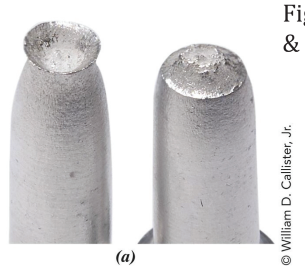


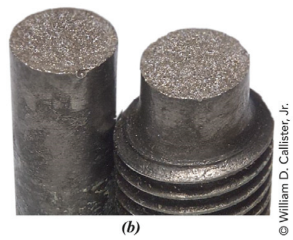
Fig. 10.1, *Callister & Rethwisch 9e.*

%*AR* or %*EL* Large Moderate Small

Fig. 10.3, Callister & Rethwisch 9e.



Cup-and-cone fracture in aluminum



Brittle fracture in mild steel

- Ductile fracture is usually more desirable than brittle fracture!
- Ductile fracture is accompanied by significant plastic deformation.
- Ductile fracture: warning before fracture (brittle fracture: no warning)

Moderately Ductile Fracture (Rupture)

- **Stages in the cup-and-cone fracture**

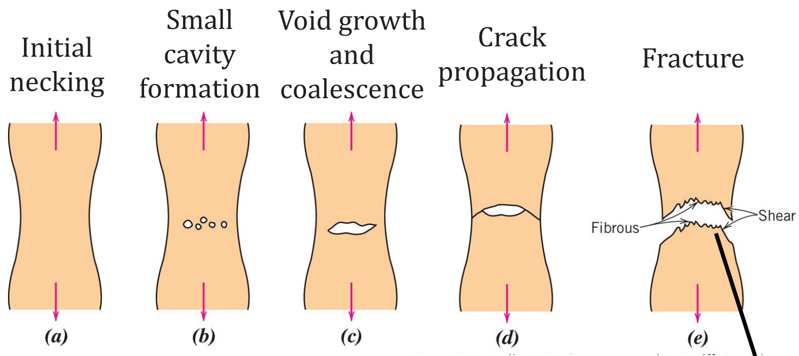
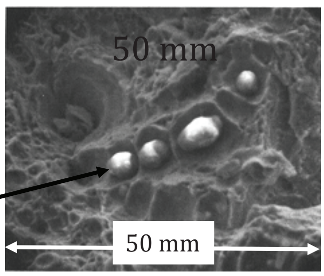


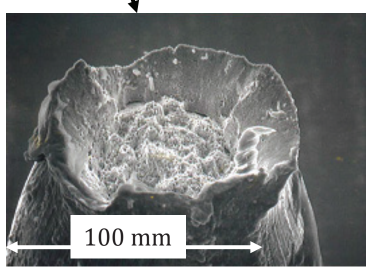
Fig. 10.2, *Callister & Rethwisch 9e.*

Resulting fracture surfaces (steel)

Particles serve as void nucleation sites.



From V.J. Colangelo and F.A. Heiser, *Analysis of Metallurgical Failures* (2nd ed.), Fig. 11.28, p. 294, John Wiley and Sons, Inc., 1987.

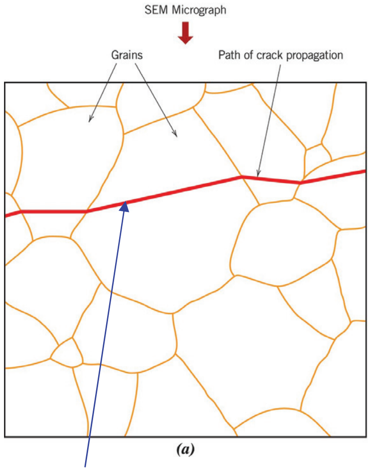


Fracture surface of tire cord wire loaded in tension. Courtesy of F. Roehrig, CC Technologies, Dublin, OH.

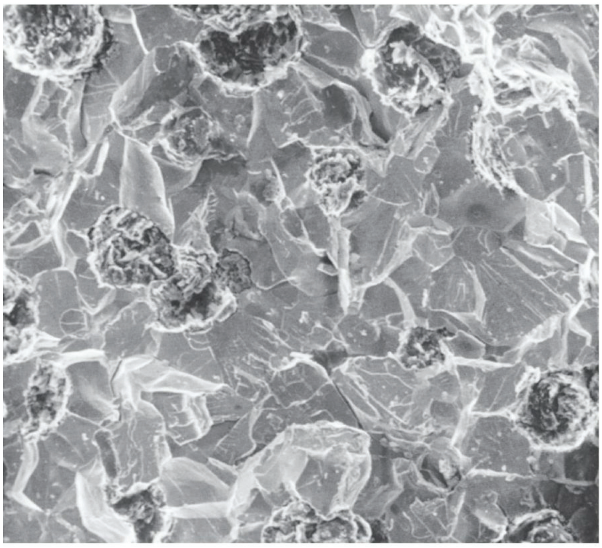
10.4 Brittle Fracture

- **Brittle Fracture**
  - Little or no plastic deformation
  - Catastrophic

I. Transgranular Fracture (Cleavage Fracture)



Cleavage within a single grain



(b)

Figure (b) from V. J. Colangelo and F. A. Heiser, Analysis of Metallurgical Failures, 2nd edition. Copyright © 1987 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.

Fig. 10.6, *Callister & Rethwisch 9e.*

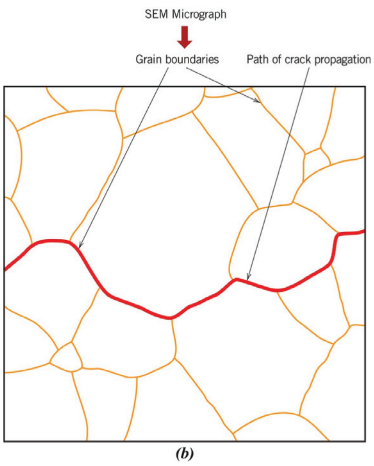
II. Intergranular Fracture

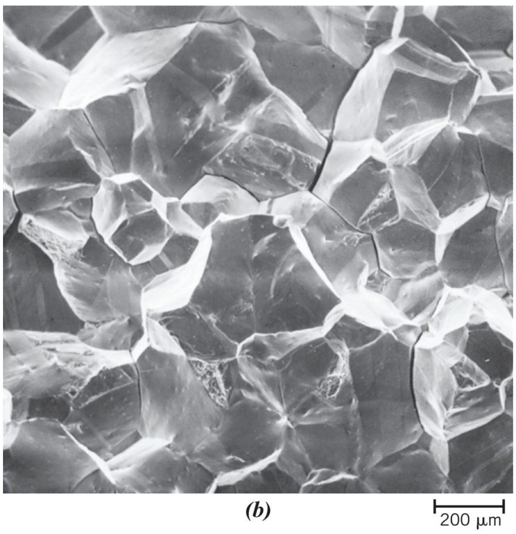
II. Intergranular Fracture:

A. Less energy absorbed than in cleavage because yield is not necessary

B. Caused by degradation of grain boundaries

1. Segregation of tramp impurities
2. Stress corrosion cracking
3. Hydrogen embrittlement





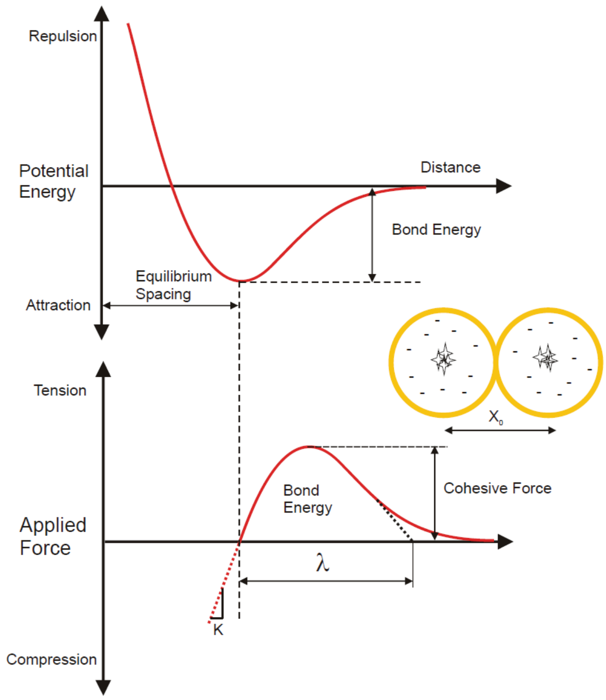
(b)

Figure (b) reproduced with permission from ASM Handbook, Vol. 12, Fractography, ASM International, Materials Park, OH, 1987.

Fig. 10.7, *Callister & Rethwisch 9e.*

10.5 Principle of Fracture Mechanics

- An Atomic View of Fracture



Bonding energy: $E_b = \int_{x_0}^{\infty} P dx$

Interatomic force-displacement relation:

$$P = P_c \sin\left(\frac{\pi x}{\lambda}\right)$$

For small displacements, force-displacement relationship is linear:

$$P = P_c\left(\frac{\pi x}{\lambda}\right) = kx, \text{ where } k = \frac{\pi P_c}{\lambda} \quad \left(E = \frac{\pi \sigma_c x_0}{\lambda}\right)$$

$$\therefore \sigma_c \approx \frac{E}{\pi}$$

Surface energy:

$$\gamma_s = \frac{1}{2}\int_0^{\lambda} \sigma_c \sin\left(\frac{\pi x}{\lambda}\right) dx = \sigma_c \frac{\lambda}{\pi}$$

$$\therefore \sigma_c = \sqrt{\frac{E\gamma_s}{x_0}}$$

Stress Concentration Effect of Flows

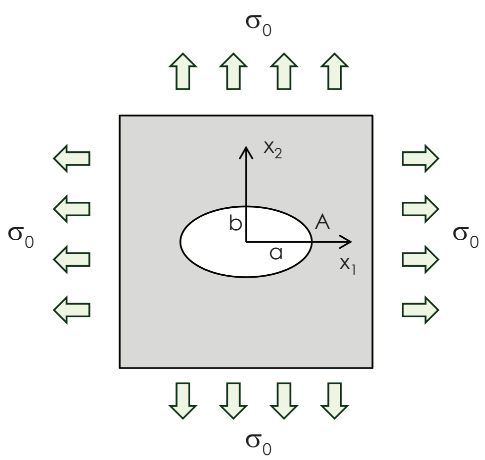
- Stress concentration around an elliptic hole by C. E. Inglis (1913)

  - Equation of an ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
  - Radius of curvature at the point A: $\rho = \frac{b^2}{a}$

$$\sigma_{yy}^{MAX}(A) = \sigma_0\left(2\frac{a}{b} + 1\right)$$

$$= \sigma_0\left(2\frac{a}{\sqrt{\rho a}} + 1\right) = \sigma_0\left(2\sqrt{\frac{a}{\rho}} + 1\right)$$

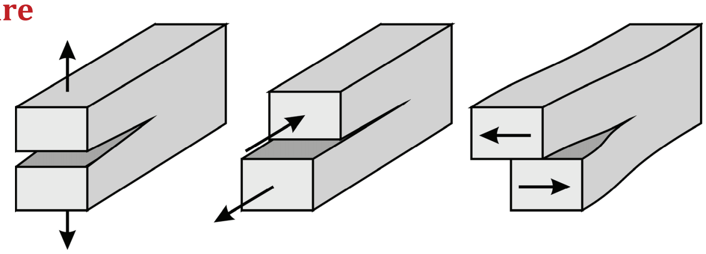
$$\cong 2\sigma_0\sqrt{\frac{a}{\rho}}$$



- The above equation show that as b → 0 (the ellipse becomes a crack) a stress singularity (σ ~ 1/√r) develops at the crack tip.

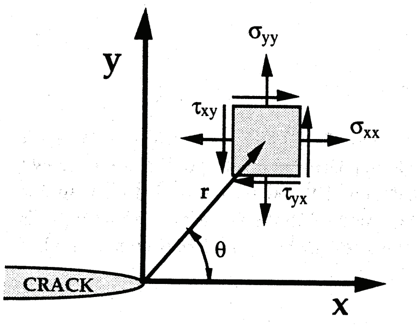
Stress Analysis of Cracks

- **Three Modes of Fracture**



Mode I (Opening) Mode II (Sliding) Mode III (Tearing)

- **Stress fields near a crack tip**



$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}^{I}(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} f_{ij}^{II}(\theta) + \frac{K_{III}}{\sqrt{2\pi r}} f_{ij}^{III}(\theta) + \sum_{m=0}^{\infty} A_m r^{\frac{m}{2}} g_{ij}^{(m)}(\theta)$$

σ_{ij} : stress tensor

K_I: Mode I stress intensity factor

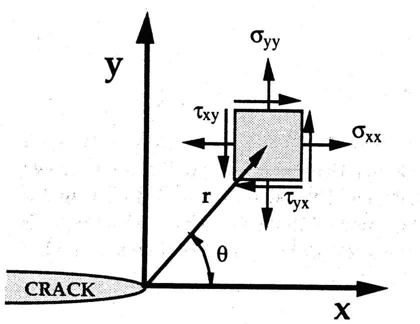
K_{II}: Mode II stress intensity factor

K_{III}: Mode III stress intensity factor

f_{ij}^{I}, f_{ij}^{II}, and f_{ij}^{III} : dimensionless functions of θ

Stress Fields near a Crack Tip

- **Stress fields near a crack tip**



$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{Bmatrix} = \frac{K_I}{\sqrt{2\pi r}}\cos(\theta/2)\begin{Bmatrix}1-\sin(\theta/2)\sin(3\theta/2)\\ 1+\sin(\theta/2)\sin(3\theta/2)\\ \sin(\theta/2)\cos(3\theta/2)\end{Bmatrix}$$

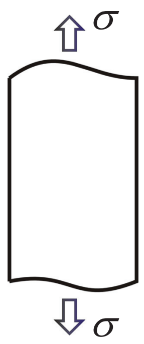
$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{Bmatrix} = \frac{K_{II}}{\sqrt{2\pi r}}\begin{Bmatrix}-\sin(\theta/2)\left[2+\cos(\theta/2)\cos(3\theta/2)\right]\\ \sin(\theta/2)\cos(\theta/2)\cos(3\theta/2)\\ \cos(\theta/2)\left[1-\sin(\theta/2)\sin(3\theta/2)\right]\end{Bmatrix}$$

$$\begin{Bmatrix}\sigma_{31}\\ \sigma_{32}\end{Bmatrix} = \frac{K_{III}}{\sqrt{2\pi r}}\begin{Bmatrix}-\sin(\theta/2)\\ \cos(\theta/2)\end{Bmatrix}$$

$$\begin{cases}K_I\text{: Mode I stress intensity factor}\\ K_{II}\text{: Mode II stress intensity factor}\\ K_{III}\text{: Mode III stress intensity factor}\end{cases}$$

Design Criteria

- **Stress approach**
  - Not consistent
  - Hardly applied to complex structures

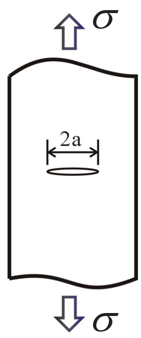


$$\sigma < \sigma_u$$

σ : applied stress

σ_u : ultimate stregth

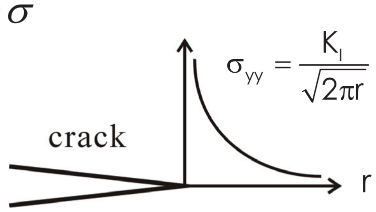
- **Fracture mechanics approach**
  - Consistent
  - Easily applied even to complex structure



$$K_I < K_{IC}$$

K_I: stress intensity factor (SIF) (calculated value) due to loading

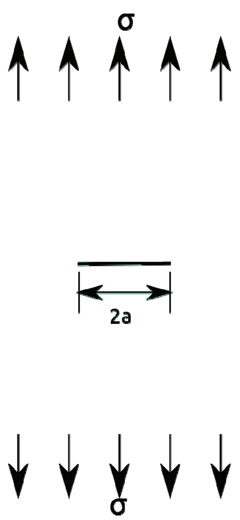
K_{IC}: fracture toughness (material property)



Examples of Stress Intensity Factors

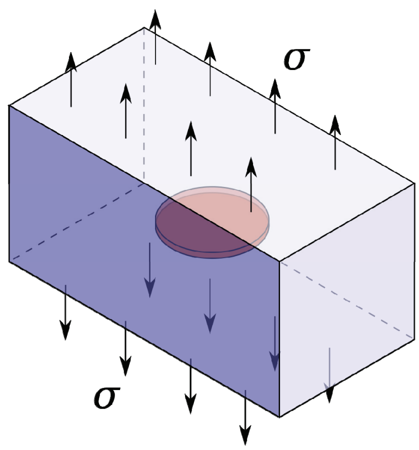
- **A centered crack in an infinite plate: under uniform uniaxial stress**

$$K_I = \sigma\sqrt{\pi a}$$



- **A penny-shaped crack in an infinite domain**

$$K_I = 2\sigma\sqrt{\frac{a}{\pi}}$$



Elastic Energy

- **Strain energy density:** $u = \int_0^{\varepsilon} \sigma_{ij} d\varepsilon_{ij}$

- **Internal energy of a deformable body:** $U = \iiint_V u dV = \iiint_V \left\{ \int_0^{\varepsilon} \sigma_{ij} d\varepsilon_{ij} \right\} dV$

- **Linear elastic materials (Hooke's law):**

$$\varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\delta_{ij}\sigma_{kk} \quad \textbf{or} \quad \sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk}$$ **(μ and λ: Lamé constant)**

- **Strain energy density of linear elastic materials:** $u = \int_0^{\varepsilon} \sigma_{ij} d\varepsilon_{ij} = \frac{1}{2}\sigma_{ij}\varepsilon_{ij}$

- **Internal energy of linear elastic materials:** $U = \iiint_V \left\{ \int_0^{\varepsilon} \sigma_{ij} d\varepsilon_{ij} \right\} dV = \frac{1}{2} \iiint_V \sigma_{ij}\varepsilon_{ij} dV$

Conservation of Energy

- Physically, the external loads tends to deform the body so that the loads do external work, W, as they are displaced.

- The external work caused by the loads is transformed into internal work or strain energy, U, which is stored within the body.

- When the loads are removed, the strain energy restores the body back to its original un-deformed position, provided that the material's elastic limit is not exceeded.

- The conservation of energy is stated as W = U

Energy Balance During Crack Growth

- The first law of thermodynamics (Law of conservation of energy)

$$\dot{W} = \dot{U}_E + \dot{U}_P + \dot{\Gamma}$$

$W:$ Work done by the applied load

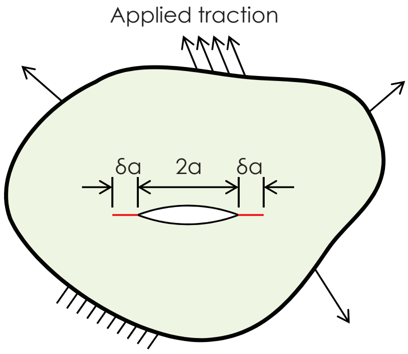
$U_E:$ Elastic energy

$U_P:$ Plastic energy

$\Gamma:$ Surface energy

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial A}\frac{\partial A}{\partial t} = \dot{A}\frac{\partial}{\partial A}$$

$$-\frac{\partial \Pi}{\partial A} = \frac{\partial U_P}{\partial A} + \frac{\partial \Gamma}{\partial A} \quad \text{where} \quad \Pi = U_E - W$$



- **The reduction of potential energy is equal to the energy dissipated in plastic work and surface creation.**
- For brittle materials, $U_P \approx 0$.

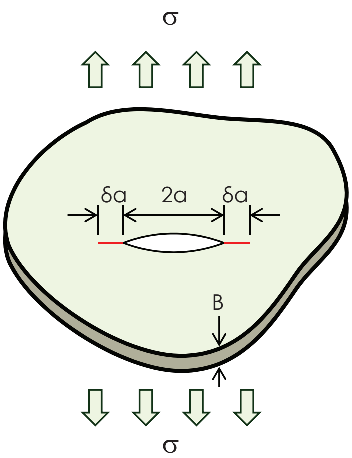
$$-\frac{\partial \Pi}{\partial A} = \frac{\partial \Gamma}{\partial A} = 2\gamma_S$$

Griffith Energy Balance

- Griffith (1920) used the stress analysis of Inglis (1913) to show

$$\Pi = \Pi_0 - \frac{\pi\sigma^2 a^2 B}{E} \quad \rightarrow \quad -\frac{\partial \Pi}{\partial A} = \frac{\pi\sigma^2 a}{E}$$

$$\Gamma = 4aB\gamma_S \quad \rightarrow \quad \frac{\partial \Gamma}{\partial A} = 2\gamma_S$$



- **Griffith fracture stress**

$$\sigma_f = \left(\frac{2E\gamma_S}{\pi a}\right)^{1/2}$$

- Modified Griffith fracture stress

$$\sigma_f = \left(\frac{2E(\gamma_P + \gamma_S)}{\pi a}\right)^{1/2}$$

$\gamma_P:$ plastic work per unit area of surface

Energy Release Rate

- **Energy release rate, G: A measure of the energy available for an increment of crack extension (Irwin, 1948)**

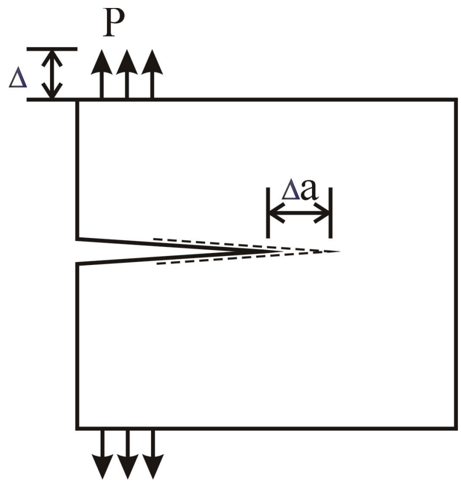
$$G = -\frac{\partial \Pi}{\partial A}$$

- **It is also called the *crack extension force* or the *crack driving force*.**

- **From the Griffith energy balance, the crack extension occurs when G reaches a critical value, i.e.,**

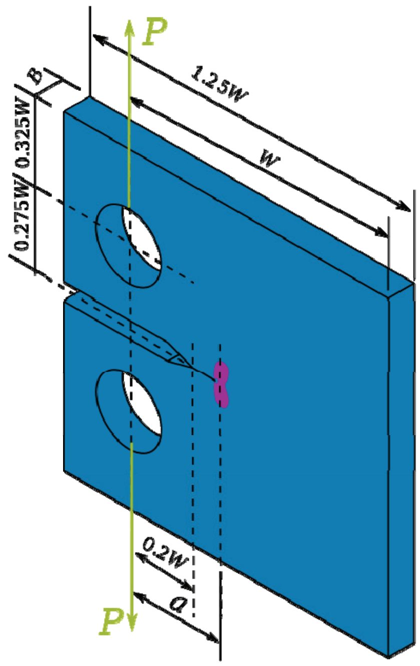
$$G\left(= -\frac{\partial \Pi}{\partial A}\right) = \frac{\pi \sigma^2 a}{E} = G_c = \frac{\partial \Gamma}{\partial A} = 2\gamma_s,$$

where G_c is a measure of the *fracture toughness* of the material.



10.6 Fracture Toughness Testing

- **Standard: ASTM-E399, D5045**
  Specimen: Compact Tension (CT)



- **Calculation**

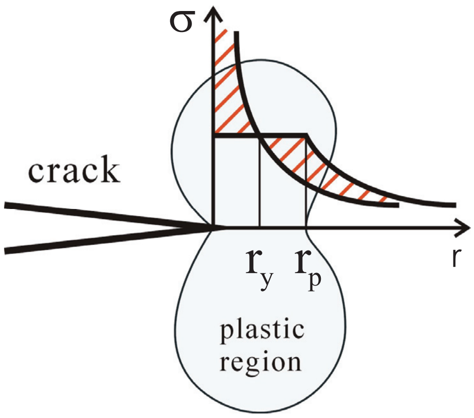
$$K_{IC} = f(x)\frac{P_Q}{BW^{1/2}}$$

K_{IC} : Fracture toughness
P_Q : Critical load

f(x) = 8.34 at x = 0.45
9.66 at x = 0.50
11.35 at x = 0.55
x = a/W, (0.45 < x < 0.55)

$$B, a \geq 2.5\left(\frac{K_{IC}}{\sigma_{yield}}\right)^2$$

Plastic Effect on Crack Tip



- Yielding zone size: $r_y = \dfrac{1}{2\pi}\left(\dfrac{K_I}{\sigma_y}\right)^2$
- Plastic zone size

$$r_p = 2r_y = \frac{1}{\pi}\left(\frac{K_I}{\sigma_y}\right)^2 \quad \text{for plane stress}$$

$$r_p = 2r_y = \frac{1}{3\pi}\left(\frac{K_I}{\sigma_y}\right)^2 \quad \text{for plane strain}$$

- Ductile fracture : sufficient plastic deformation before fracture
- Brittle fracture : small plastic deformation before fracture

3D Aspects of Plastic Zone

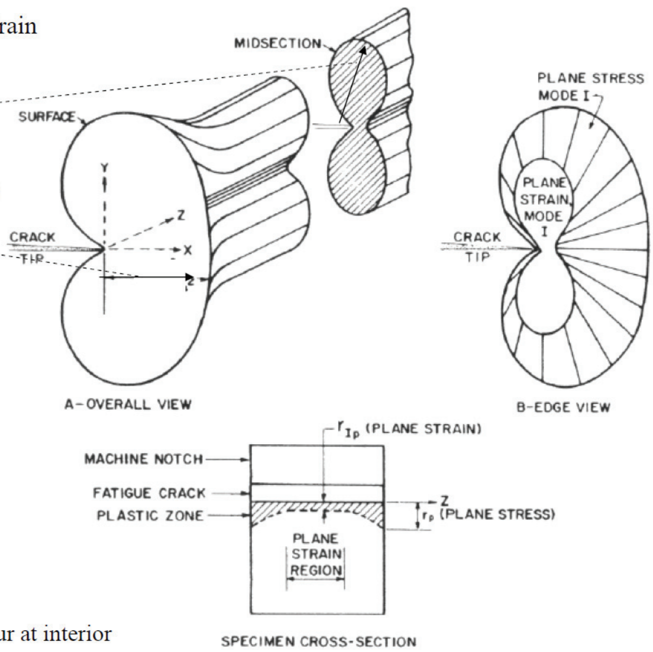
3-D Aspects of the Plastic Zone at Crack Tip for Mode I, Small Scale Yielding

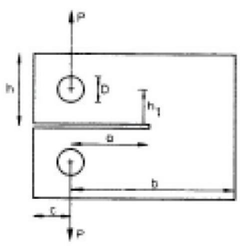
Recall, for mode I plane strain in small scale yielding

$$r_p \cong \frac{1}{3\pi}\left(\frac{K}{\sigma_Y}\right)^2$$

and for mode I plane stress

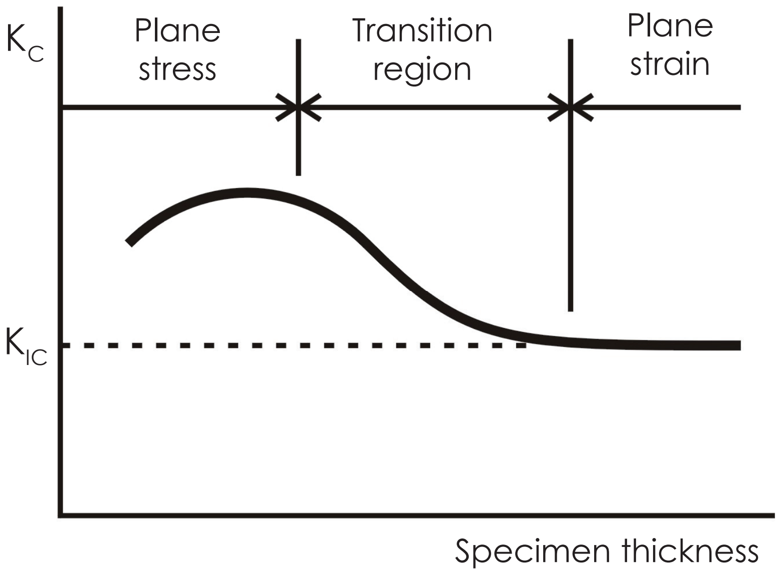
$$r_p \cong \frac{1}{\pi}\left(\frac{K}{\sigma_Y}\right)^2$$



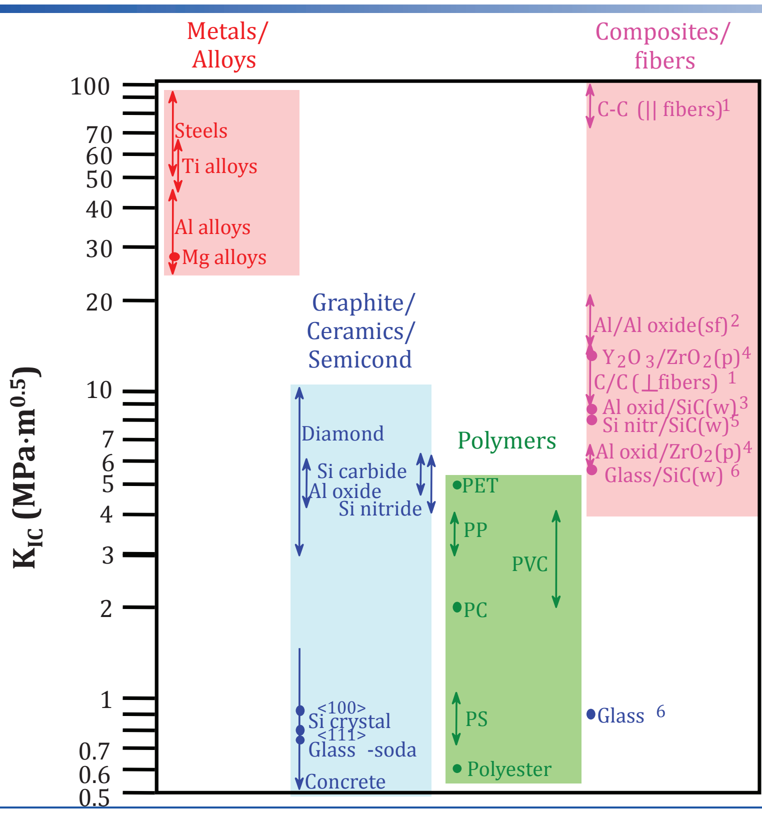


Plane strain ssy conditions occur at interior of crack tip if r_p << thickness

Variation of K_C with Specimen Thickness



Fracture Toughness Ranges



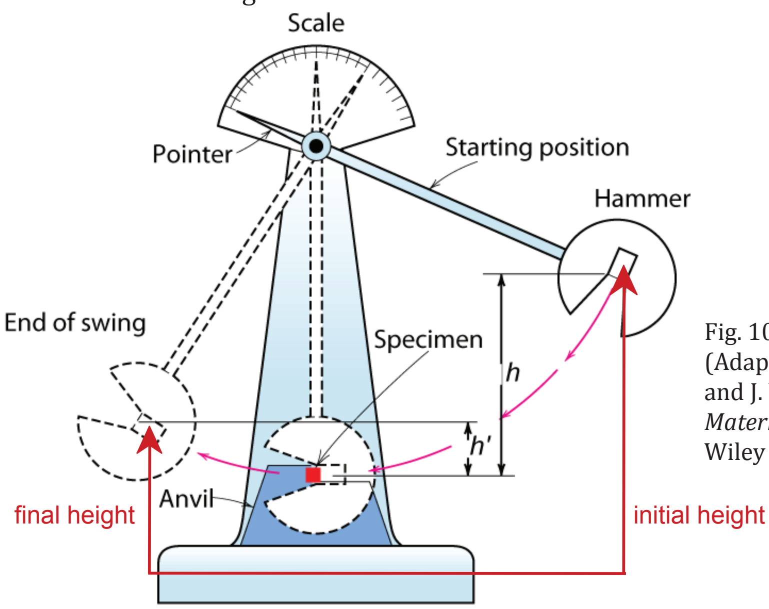
Based on data in Table B.5,
Callister & Rethwisch 9e.

Composite reinforcement geometry is: f = fibers; sf = short fibers; w = whiskers; p = particles. Addition data as noted (vol. frac tion of reinforcement):

1. (55vol%) *ASM Handbook*, Vol. 21, ASM Int., Mat erials Park, OH (2001) p. 606.
2. (55 vol%) Courtesy J. Cornie, MMC, Inc., Waltha m, MA.
3. (30 vol%) P.F. Becher et al., *Fracture Mechanics of Ceramics*, Vol. 7, Plenum Press (1986). pp. 61-7 3.
4. Courtesy CoorsTek, Golden, CO.
5. (30 vol%) S.T. Buljan et al., "Development of Ce ramic Matrix Composites for Application in Techn ology for Advanced Engines Program", ORNL/Sub /85-22011/2, ORNL, 1992.
6. (20vol%) F.D. Gace et al., *Ceram. Eng. Sci. Proc.*, Vol. 7 (1986) pp. 978-82.

Impact Testing

- **Impact loading:**
  - severe testing case
  - makes material more brittle
  - decreases toughness



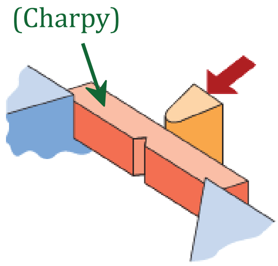
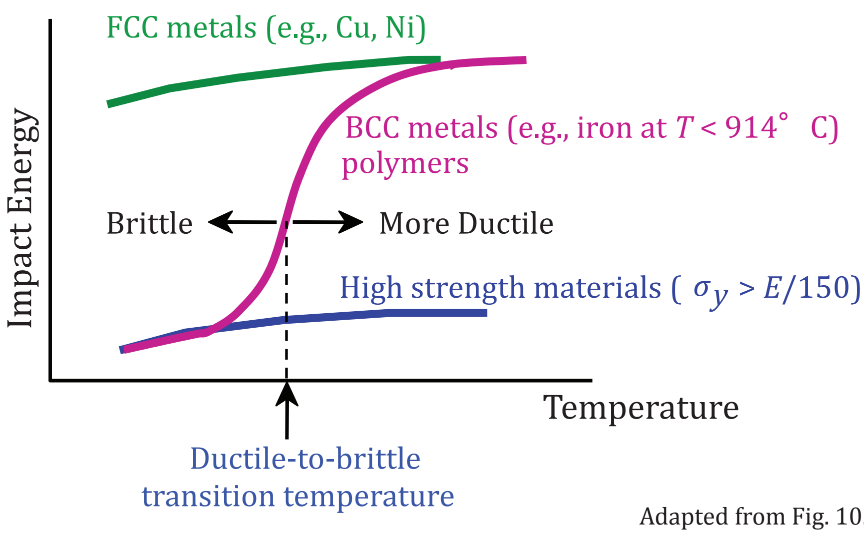


Fig. 10.12(b), *Callister & Rethwisch 9e.* (Adapted from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, John Wiley and Sons, Inc. (1965) p. 13.)

Influence of Temperature on Impact Energy

- **Ductile-to-Brittle Transition Temperature (DBTT)...**



Adapted from Fig. 10.15, *Callister & Rethwisch 9e.*

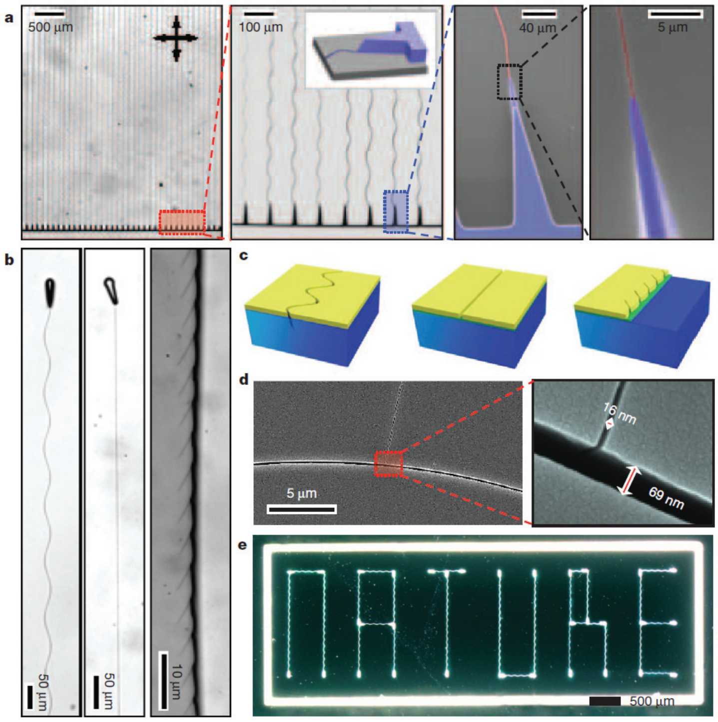
Liberty Ship during World War II

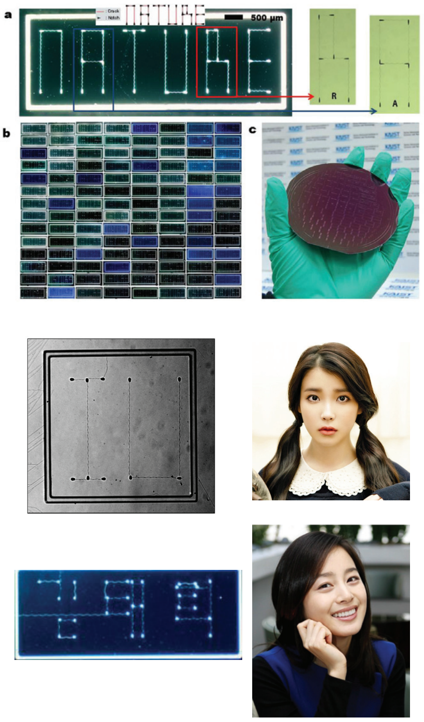
Liberty Ship during World War II

- Liberty Ship: http://en.wikipedia.org/wiki/Liberty_ship
- Early Liberty ships suffered hull and deck cracks, and a few were lost to such structural defects. During World War II, there were nearly 1,500 instances of significant brittle fractures. Twelve ships, including three of the 2,710 Liberties built, broke in half without warning, including the *SS John P. Gaines*,[10][11] which sank on 24 November 1943 with the loss of 10 lives. Suspicion fell on the shipyards which had often used inexperienced workers and new welding techniques to produce large numbers of ships in great haste.[12] The Ministry of War Transport lent the British-built *Empire Duke* for testing purposes.[13] Constance Tipper of Cambridge University demonstrated that the fractures were not initiated by welding, but instead by the grade of steel used, which suffered from embrittlement.[12] She discovered that the ships in the North Atlantic were exposed to temperatures that could fall below a critical point when the mechanism of failure changed from ductile to brittle, and thus the hull could fracture rather easily. The predominantly welded (as opposed to riveted) hull construction then allowed cracks to run for large distances unimpeded. One common type of crack nucleated at the square corner of a hatch which coincided with a welded seam, both the corner and the weld acting as stress concentrators. Furthermore, the ships were frequently grossly overloaded and some of the problems occurred during or after severe storms at sea that would have placed any ship at risk. Various reinforcements were applied to the Liberty ships to arrest the crack problems, and the successor design, the Victory ship, was stronger and less stiff to better deal with fatigue.

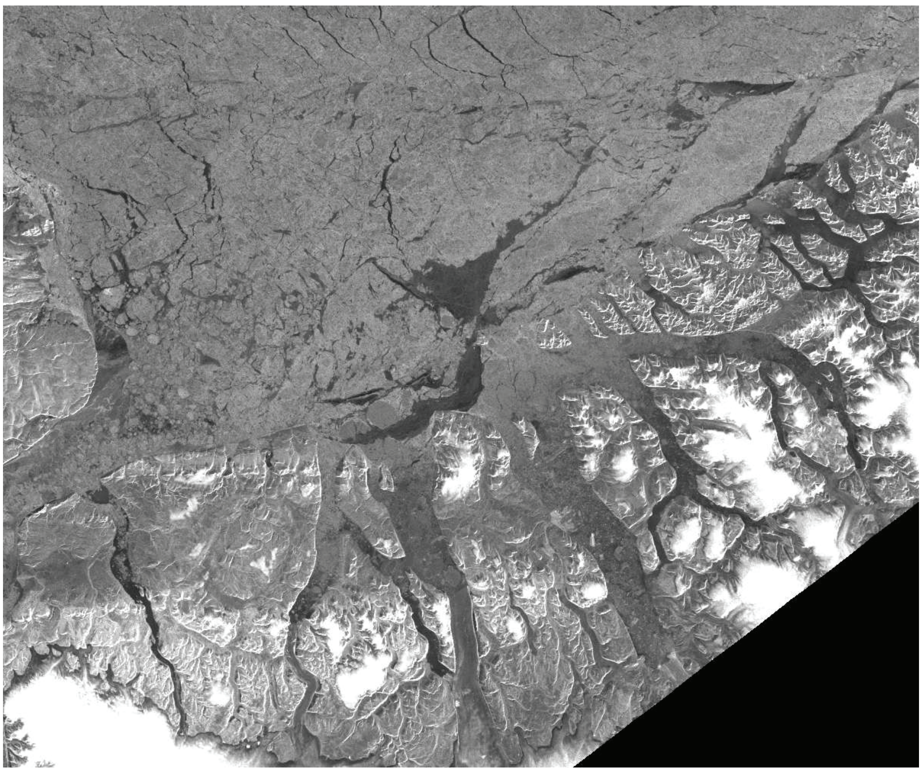
Patterning by Controlled Cracking

K. H. Nam, I. H. Park, & S. H. Ko, Patterning by controlled cracking, Nature, Vol. 485, pp. 221-224, 2012.



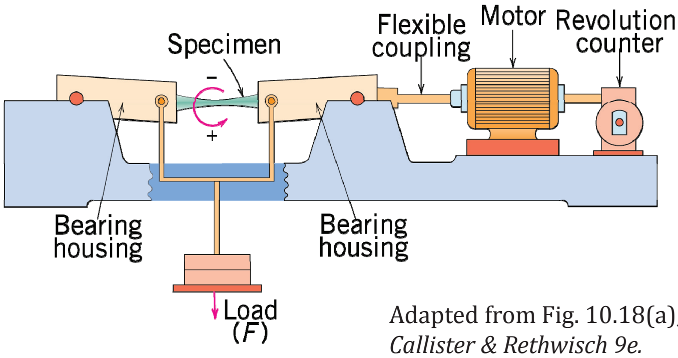


Fragmentation of Ice in the Arctic Ocean



10.7 Cyclic Stresses-Fatigue

- **Fatigue = failure under applied cyclic stress.**



Adapted from Fig. 10.18(a), *Callister & Rethwisch 9e.*

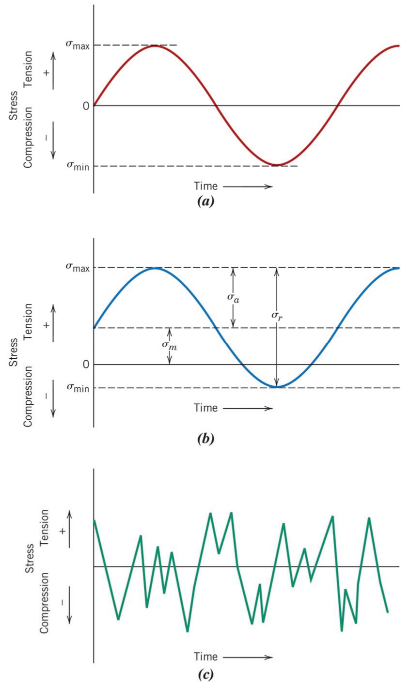


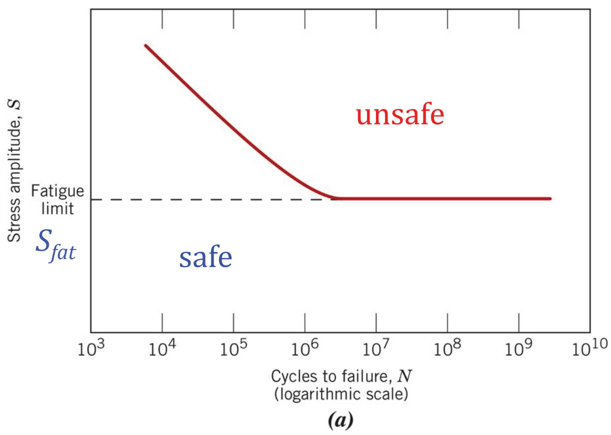
Fig. 10.17, *Callister & Rethwisch 9e.*

- **Stress varies with time.**
  - key parameters are S, σm, and cycling frequency

- **Key points: Fatigue...**
  - can cause part failure, even though $\sigma_{max} < \sigma_y$.
  - responsible for ~ 90% of mechanical engineering failures.

10.8 The S-N Curve

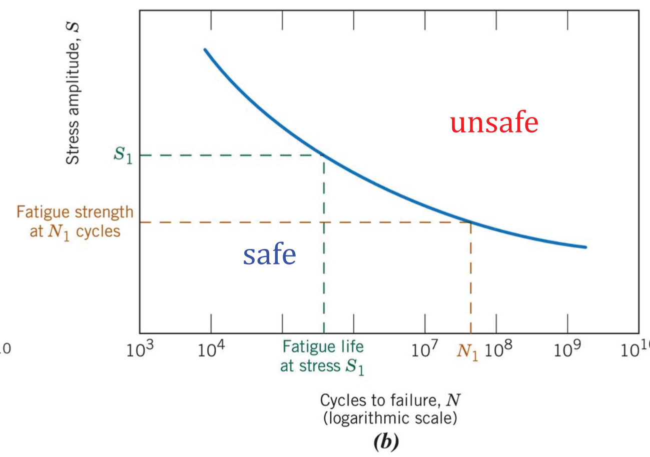
- **Fatigue limit, S_{fat}:**
  - no fatigue failure if $S < S_{fat}$



Case for Steel (typical)

Adapted from Fig. 10.19(a), *Callister & Rethwisch 9e.*

- **For some materials,**
  - there is no fatigue limit!



Case for Al (typical)

Adapted from Fig. 10.19(b), *Callister & Rethwisch 9e.*

10.9 Crack Initiation and Propagation

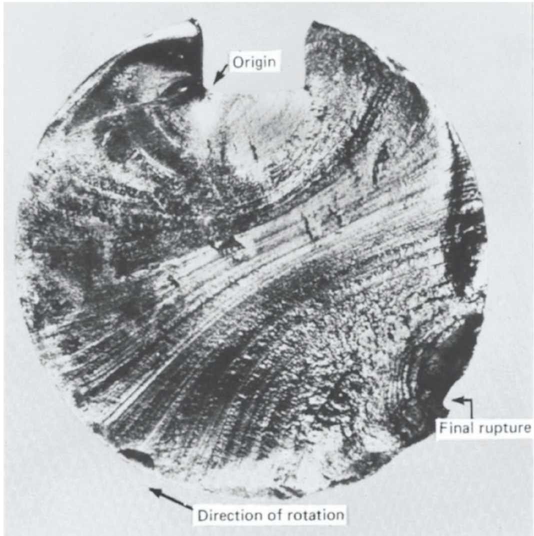
- **Rate of Fatigue Crack Growth: Crack grows incrementally**

$$\frac{da}{dN} = (\Delta K)^m$$

typically 1 to 6

$$\sim (\Delta\sigma)\sqrt{a}$$

increase in crack length per loading cycle



From D. J. Wulpi, Understanding How Components Fail, 1985.
Reproduced by permission of ASM International, Materials Park, OH.

- **Failed rotating shaft**
  - crack grew even though $K_{max} < K_{IC}$
  - crack grows faster as
    - Δσ increases
    - crack gets longer
    - loading freq. increases.

Fig. 10.22, *Callister & Rethwisch 9e.*
(From D. J. Wulpi, *Understanding How Components Fail*, 1985.
Reproduced by permission of ASM International, Materials Park, OH.)

10.10 Factors That Affect Fatigue Life

1. **Impose compressive surface stresses**
   (to suppress surface cracks from growing)

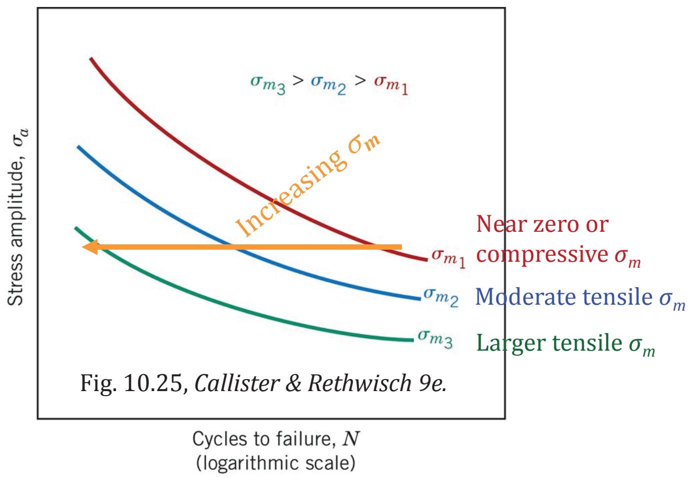
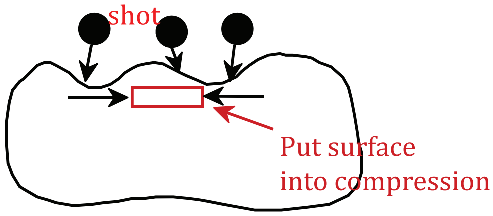
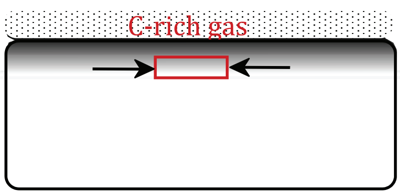


Fig. 10.25, *Callister & Rethwisch 9e.*

-Method 1: shot peening



-Method 2: carburizing



2. Remove stress concentrators.

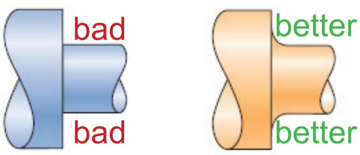
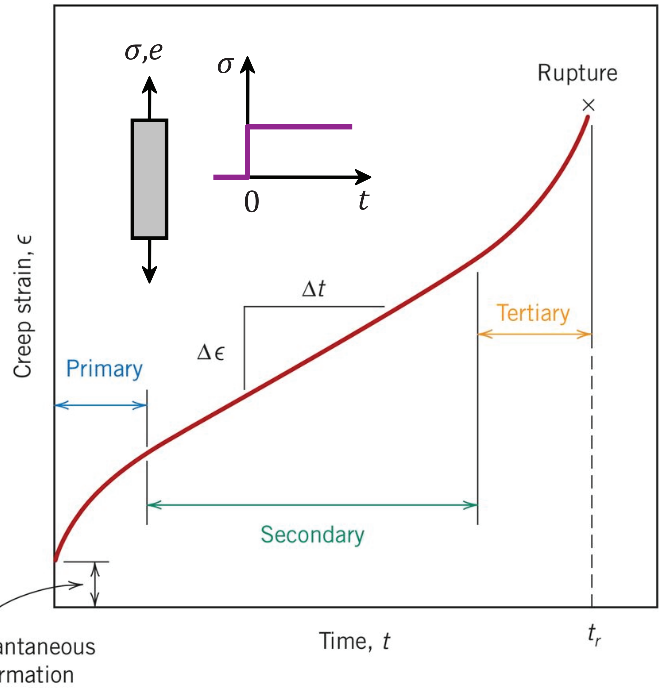


Fig. 10.26, *Callister & Rethwisch 9e.*

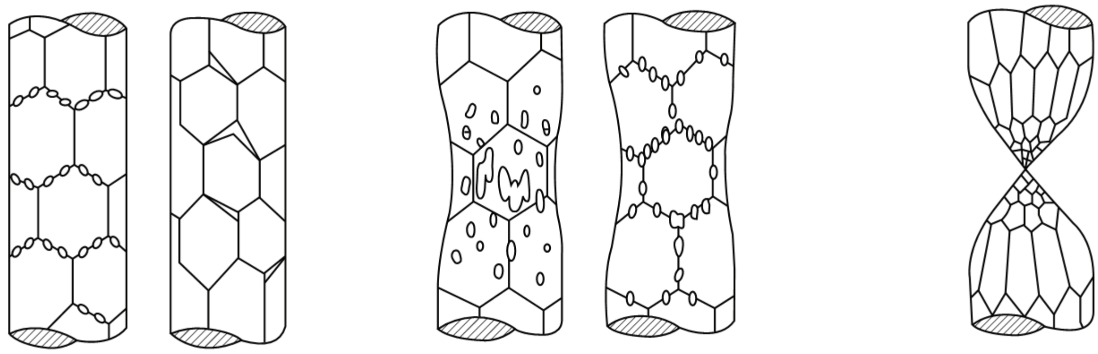
10.12 Generalized Creep Behavior

- **Creep phenomenon: Time-dependent and permanent deformation of materials when subjected to constant load or stress.**
  - Instantaneous deformation
  - Primary Creep Stage or Transient Creep: slope (creep rate) decreases with time.
  - Secondary Creep Stage: steady-state, i.e., constant slope ($\Delta e/\Delta t$)
  - Tertiary Creep Stage: or Accelerated Creep: slope (creep rate) increases with time.



Creep Fracture Mechanism

- **Schematic drawing of three fracture mechanisms in a high temperature creep regime**



Intergranular creep fracture
(voids) (wedge cracks)

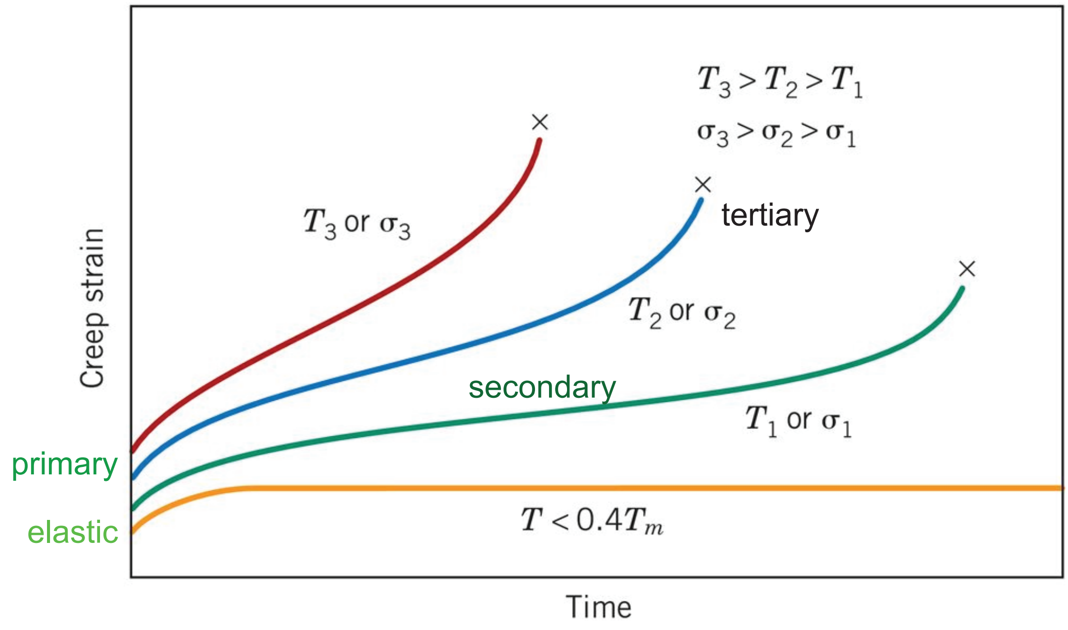
Growth of voids by power-law creep
(transgranular) (intergranular)

Rupture due to dynamic recovery or recrystallization

[Abe et al., Creep-Resistant Steels, 2008]

10.13 Stress and Temperature Effects

- Occurs at elevated temperature, $T > 0.4\ T_m$ (in K)



Figs. 10.30, *Callister & Rethwisch 9e.*

Secondary Creep

- **Strain rate is constant at a given (T, s)**
  - strain hardening is balanced by recovery

stress exponent (material parameter)

$$\dot{\varepsilon}_s = K_2 \sigma^n \exp\left(-\frac{Q_c}{RT}\right)$$

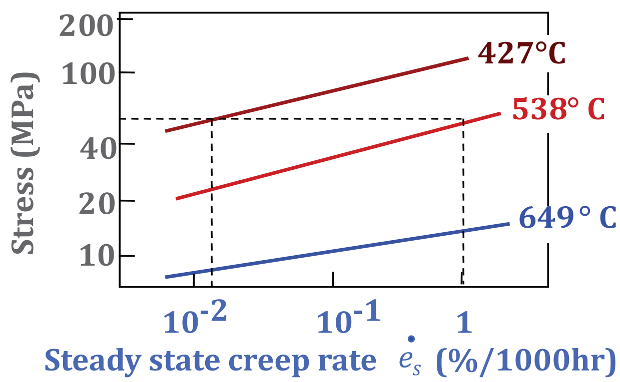
strain rate

material const.

applied stress

activation energy for creep (material parameter)

- **Strain rate increases with increasing T, σ**



Adapted from Fig. 9.38, *Callister & Rethwisch 4e.* [Reprinted with permission from *Metals Handbook: Properties and Selection: Stainless Steels, Tool Materials, and Special Purpose Metals*, Vol. 3, 9th ed., D. Benjamin (Senior Ed.), ASM International, 1980, p. 131.]

Arrhenius Equation

- **The Arrhenius equation (Svante Arrhenius, 1889)**
  - A formula for the temperature dependence of reaction rates.
  - For many common chemical reactions at room temperature, the reaction rate doubles for every 10 degree Celsius increase in temperature.
  - It can be used to model the temperature variation of diffusion coefficients, population of crystal vacancies, creep rates, and many other thermally-induced processes/reactions.

$$k = A\exp\left(-\frac{E_a}{RT}\right) \quad \text{or} \quad k = A\exp\left(-\frac{E_B}{k_B T}\right)$$

where

- k = the rate constant
- T = the absolute temperature (in kelvins)
- A = the pre-exponential factor, a constant for each chemical reaction
- E_a = the activation energy for the reaction (in Joules mol^{-1})
- R = the universal gas constant
- E_B = the activation energy for the reaction (in Joules)
- k_B = the Boltzmann constant

10.14 Data Extrapolation Methods

- **The Larson–Miller parameter is a means of predicting the lifetime of material vs. time and temperature using a correlative approach based on the Arrhenius rate equation.**
- **Larsen Miller parameter P_{LM} is used to represent creep-stress rupture data.**

$$\dot{\varepsilon}_s = K_2\sigma^n \exp\left(-\frac{Q_c}{RT}\right)$$

$$\frac{\Delta l}{\Delta t} = A'\exp\left(-\frac{Q_c}{RT}\right)$$

$$\ln\left(\frac{\Delta l}{\Delta t}\right) = \ln(A') - \frac{Q_c}{RT}$$

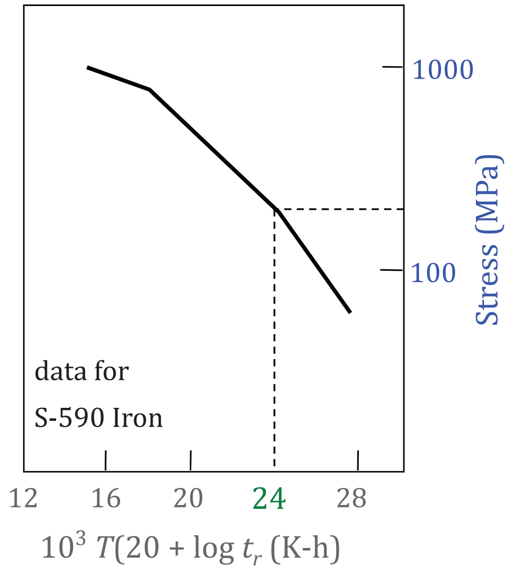
$$\frac{Q_c}{R} = T[B + \ln(\Delta t)] \quad \text{where} \quad B = \ln\left(\frac{A'}{\Delta l}\right)$$

$$\therefore \; P_{LM} = T[C + \log(t_r)]$$

- T = temperature(K),
- t_r = stress-rupture time (h)
- C = Constant (order of 20)

Prediction of Creep Rupture Lifetime

- Estimate rupture time: S-590 Iron, T = 800°C, s = 140 MPa



Adapted from Fig. 10.33, *Callister & Rethwisch 9e.* (From F.R. Larson and J. Miller, *Trans. ASME*, **74**, 765 (1952). Reprinted by permission of ASME)

Time to rupture, t_r

$$T(20+\log t_r) = P_{LM}$$

temperature

time to failure (rupture)

function of applied stress

$$(1073K)(20+\log t_r) = 24x10^3$$

Ans: t_r = 233 hr

SUMMARY

- **Engineering materials not as strong as predicted by theory**
- **Flaws act as stress concentrators that cause failure at stresses lower than theoretical values.**
- **Sharp corners produce large stress concentrations and premature failure.**
- **Failure type depends on T and σ:**
  - For simple fracture (noncyclic σ and T < $0.4T_m$), failure stress decreases with:
    - increased maximum flaw size,
    - decreased T,
    - increased rate of loading.
  - For fatigue (cyclic σ): cycles to fail decreases as Δσ increases.
  - For creep (T > $0.4T_m$): time to rupture decreases as σ or T increases.